BASED ON HEXAGONAL FUZZY NUMBER A NEW EXTENSION OF FUZZY DCEISION BY OPINION SCORE METHOD

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Abstract:
One of the most crucial concepts in operations research and the expert system is multi-criteria decision-making (MCDM) , which contains several alternatives and decision criteria . The aim of multicriteria decision making is to identify the best alternative among several alternatives that satisfy the specified criteria. Several approaches, such as TOPSIS and analytical hierarchy process (AHP) have been suggested for handling MCDM problems. These are classical MCDM approaches that calculate the alternative weights and ratings of the criteria in crisp numbers based on the opinions/preferences of the expert. In real life, the expert judgement may be incorrect due to the uncertainty of human judgement. To deal with imprecise information (Zadeh 1965) proposed using the fuzzy set theory as a modelling tool for complex systems. Fuzzy sets were first used in the area of of MCDM by (Zimmermann 1978) and (Zadeh and Bellman 1970). They paved the way for a new family of ways to deal with issues that cannot be resolved using conventional MCDM techniques, which are known as fuzzy MCDM (FMCDM), i.e., fuzzy TOPSIS ,fuzzy AHP, fuzzy ANP… etc. recently fuzzy decision by opinion score method (FDOSM) presented To solve different issues, Such as inconsistency, vagueness, and the problem of criteria weighting. Several developments have been made on this method using various kinds of fuzzy numbers in order to solve the problems that this method suffers from, the latest of which is this work using hexagonal-fuzzy numbers to address the problem of uncertainty, which is one of the main problems that researchers face in this field.
Keywords: multi criteria decision-making; MCDM; Hexagonal fuzzy number; FDOSM

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1. Introduction

MCDM is a core part of expert systems and modern decision science, involving multiple decision criteria and numerous decision alternatives [1]. Its roots of it go back to (the early 1930s) [2]. The core goal of MCDM is to select the best alternative from a collection of alternatives that meet decision-maker's needs and preferences based on their multiple criteria [1] [3]. There are two types of general MCDM problems: those with a finite and an infinite number of alternatives; it has been widely used in many fields, including The medical field [4]–[8], education field [9]–[12]. Several approaches, such as DEA , TOPSIS and AHP , have been suggested for handling MCDM problems. These are classical MCDM approaches that measure the alternative weights and ratings of the criteria in crisp numbers based on the opinions/preferences of the expert. In real life, the expert judgement may be incorrect due to the uncertainty of human judgement [13]. also, It is widely acknowledged that the majority of decisions made in the real world occur in environments where the aims and constraints are not precisely known due to their complexity, and hence the problem cannot be accurately defined or expressed in crisp value [14].To deal with imprecise information (Zadeh 1965) [15] proposed using the fuzzy set theory as a modelling tool for complicated systems . Fuzzy logic is type of many-valued logic; it deals with approximate logic instead of exact logic. Fuzzy logic values can have a truth value that ranges from 0 to 1, unlike conventional binary sets where values may have either false or true values [16]. Fuzzy sets have applications in artificial intelligence [17]–[19], control engineering, computer science [20]–[22], decision theory, management science, expert systems, logic, etc. [23], [24]. (Zimmermann 1978) and (Bellman and Zadeh 1970) introduced fuzzy sets into multi-criteria decision making field. They paved the way for a new family of ways to deal with issues that cannot be resolved using conventional MCDM techniques, which are known as fuzzy MCDM (FMCDM) [14], i.e., fuzzy ANP, fuzzy AHP, fuzzy TOPSIS, etc. [13]. Generally, MCDM approaches are classified into two main approaches: 1- The first strategy is the human approach, which contains the best-worst method (BWM), AHP, and the analytic network process (ANP). 2- The second strategy is the math strategy, which consists of several techniques such as (simple additive weighting (SAW) and (TOPSIS)). Each strategy addresses particular difficulties. For instance, consistency issues, unnatural comparisons, ambiguity, normalisation, measuring distance. FDOSM a new MCDM technique, is introduced to address these problems. (i)Data input, (ii)data transformation, and(iii) data processing components are the three phases of the FDOSM. [25] . Several developments have been made on this method using various kinds of fuzzy numbers in order to solve the problems that this method suffers from, the latest of which is this work using hexagonal-fuzzy number to address the issue of uncertainty, which is one of the main problems that researchers face in this field . Rajarajeshwari first described the Hexagonal-Fuzzy Number (HXFN) in 2013 [26] , is specified by six tuples W= (e, r, t, y, s, d) where (e, r, t, y, s, and d) are real numbers . This type was chosen for several reasons that characterize this type: 1- Hexagonal-fuzzy number (HXFN) is maintains the essence of a fuzzy number and minimises information loss [27] . 2- In few cases Trapezoidal or Triangular is not applicable to address the issue if it has six various points; in such cases hexagonal fuzzy number are use, also Hexagonal-fuzzy numbers (HFN) encapsulate ambiguity in a more comprehensive manner than triangular (TFN), trapezoidal (TrFN), and pentagonal (PFN) [26],[28],[29] 3- We more closely analyse the data supplied by the experts using this fuzzy number. [30].

2. Preliminaries

2.1 FDOSM

Is a novel approach to MCDM in a fuzzy environment. proposed in 2020 [25] To solve the challenges faced by each human and mathematical MCDM approaches, such as inconsistency, vagueness, unnatural comparison. [31].

The FDOSM steps can be summarised as follows: [32], [25].

Stage1: Build the decision matrix.

Stage2: Choose the optimal solution for each criterion, where the optimal solution is either minimum, maximum, or critical value.

Stage 3: Based on the expert opinion, build the opinion matrix by referencing comparisons between both the optimal solution and other values for every criterion.

Stage 4: Transform the opinion matrix into (TFNs).

Stage 5: Direct arithmetic mean aggregation.

Stage 6: The smallest alternative is the finest option.

2.2 Fuzzy sets

Fuzzy set ideas require an understanding of the basic idea of classical set theory. The idea of a classical set in mathematics is quite straightforward. A group of well-defined objects is referred to as a set. These objects are either part of the collection or they don't. the classical set A can be characterized by the function $\mu_{\mathrm{A}}(\mathrm{x})$, which takes 1 or 0, depending on whether or not the element x is in A : [33]

$$\mu_A(x) = \begin{cases} 1 & \text{for } x \in A \\ 0 & \text{for } x \notin A \end{cases} \quad (1)$$

Hence $\mu_{\mathrm{A}}(\mathrm{x}) \in \{0,1\}$, $\mu_{\mathrm{A}}(\mathrm{x})$ takes only 1 or 0.
while the concept of fuzzy set takes values in the interval [0, 1]. Represent the degree of membership.

A fuzzy set R is defined as:

$\mathrm{R} = \{(\mathrm{x}, \mu_{\mathrm{R}}(\mathrm{x}))/\mathrm{x} \in \mathrm{A}, \mu_{\mathrm{R}}(\mathrm{x}) \in [0,1]\}$ (2)

where $\mu_{\mathrm{R}}(\mathrm{x})$ is a membership function; $\mu_{\mathrm{R}}(\mathrm{x})$ specifies the degree to which each element of A belongs to the fuzzy set R. In the midst of the twentieth century, Professor Zadeh developed the concept of fuzzy sets. [34]. This theory provides a solution to the issue of uncertainty and ambiguity in computing systems that make use of linguistic and ambiguous variables. [35].

2.2.1 Hexagonal-Fuzzy Number[HFN]:

[HFN] $\tilde{F}_H$ Is a Fuzzy Number denoted by $\tilde{F}_H = (\hat{\jmath}_1, \hat{\jmath}_2, \hat{\jmath}_3, \hat{\jmath}_4, \hat{\jmath}_5, \hat{\jmath}_6)$, where $\hat{\jmath}_1, \hat{\jmath}_2, \hat{\jmath}_3, \hat{\jmath}_4, \hat{\jmath}_5$ and $\hat{\jmath}_6$ are real numbers, and ĵ1 ≤ ĵ2 ≤ ĵ3 ≤ ĵ4 ≤ ĵ5 ≤ ĵ6. We can express its membership function as follows. [30], [36]:

$$\mu_{\tilde{F}}(x) = \begin{cases} \frac{1}{2}\left(\frac{x-\hat{\jmath}_1}{\hat{\jmath}_2-\hat{\jmath}_1}\right), & \text{for } \hat{\jmath}_1 \le x \le \hat{\jmath}_2 \\ \frac{1}{2}+\frac{1}{2}\left(\frac{x-\hat{\jmath}_2}{\hat{\jmath}_3-\hat{\jmath}_2}\right), & \text{for } \hat{\jmath}_2 \le x \le \hat{\jmath}_3 \\ 1, & \text{for } \hat{\jmath}_3 \le x \le \hat{\jmath}_4 \\ 1-\frac{1}{2}\left(\frac{x-\hat{\jmath}_4}{\hat{\jmath}_5-\hat{\jmath}_4}\right), & \text{for } \hat{\jmath}_4 \le x \le \hat{\jmath}_5 \\ \frac{1}{2}\left(\frac{\hat{\jmath}_6-x}{\hat{\jmath}_6-\hat{\jmath}_5}\right), & \text{for } \hat{\jmath}_5 \le x \le \hat{\jmath}_6 \\ 0, & \text{otherwise} \end{cases} \quad (3)$$

Operation on Hexagonal-Fuzzy Numbers:

In order to specify the arithmetic operations between two HFN, let us assume that U = (Ù1, Ù2, Ù3, Ù4, Ù5, Ù6) and w = (ŵ1, ŵ2, ŵ3, ŵ4, ŵ5, ŵ6) [30],[37] :

1- Addition: $(U + w) = (\grave{U}_1 + \hat{w}_1, \grave{U}_2 + \hat{w}_2, \grave{U}_3 + \hat{w}_3, \grave{U}_4 + \hat{w}_4, \grave{U}_5 + \hat{w}_5, \grave{U}_6 + \hat{w}_6)$ (4)
2- Subtraction: $(U - w) = (\grave{U}_1 - \hat{w}_1, \grave{U}_2 - \hat{w}_2, \grave{U}_3 - \hat{w}_3, \grave{U}_4 - \hat{w}_4, \grave{U}_5 - \hat{w}_5, \grave{U}_6 - \hat{w}_6)$ (5)
3- Multiplication: $(U \times w) = (\grave{U}_1\hat{w}_1, \grave{U}_2\hat{w}_2, \grave{U}_3\hat{w}_3, \grave{U}_4\hat{w}_4, \grave{U}_5\hat{w}_5, \grave{U}_6\hat{w}_6)$ (6)
4- Division: $\left(\frac{U}{w}\right) = \left(\frac{\grave{U}_1}{\hat{w}_1}, \frac{\grave{U}_2}{\hat{w}_2}, \frac{\grave{U}_3}{\hat{w}_3}, \frac{\grave{U}_4}{\hat{w}_4}, \frac{\grave{U}_5}{\hat{w}_5}, \frac{\grave{U}_6}{\hat{w}_6}\right)$ (7)

3. Related Work

To handle a multi-criteria decision issue, numerous academic researchers have used FDOSM or expanded FDSOM into another fuzzy environment. The following are the findings from our review of all papers on FDOSM: According to the authors of [38] FDOSM expanded in the following ways: (1) Application of different aggregation methods within the direct aggregation MCDM method. (2) A discussion of the effectiveness of each kind in relation to the final AQM benchmarking. and (3) Using several MCDM methodologies on FDOSM to achieve the optimal benchmarking result for AQM methods. [39] This study has extended FDOSM and FWZIC using q-rung ortho-pair fuzzy numbers. [40] This research expanded FDOSM to a fuzzy type-2 environment that uses interval type-2 trapezoidal membership (IT2T). The researcher in [41] developed FDOSM and FWZIC techniques for the T-SFSs environment called T-SFDOSM and T-SFWZIC, so that they can be used for the distribution of COVID-19 vaccinations. [42] This study presents a new uniform Pythagorean fuzzy framework for the distribution of the COVID-19 vaccine dose. by Combining PFDOSM and a new version of the Pythagorean fuzzy weighted zero-inconsistency PFWZIC. In this study [43], a new extension of FDOSM for benchmarking and evaluating SLRSs is developed using an Interval-Valued Pythagorean Fuzzy Set IVPFS named IVP-FDOSM. [44] this study extends FWZIC and FDOSM using a neutrosophic fuzzy set to address the same issues while benchmarking the applications. [45] This paper successfully evaluated and benchmarked the real-time SLRS by extending FDOSM into the Pythagorean fuzzy set with the help of the Interaction hybrid arithmetic mean (IHAM) operator (PFDOSM-IHAM). [46] in this study FDOSM and Fuzzy weighted zero-inconsistency FWZIC are both extended based on Cubic Pythagorean fuzzy sets CPFS, called CP-FDOSM and CP-FWZIC . [47] in this study, FDOSM and FWZIC are both extended based on Pythagorean m-polar fuzzy sets . [31] this research extends FDOSM using Fermatean fuzzy sets, called Fermatean-FDOSM, so as to benchmark the real-life issues effectively. [32] In this work, the author employ 2-tuple fuzzy sets to address the loss-of-information issue by expanding the FDOSM into the 2-tuple-FDOSM. [48] this study extended FDOSM into intuitionistic FDOSM using an intuitionistic fuzzy set, to benchmark and evaluation of the efficiency and reliability of DASs systems. According to academic articles, no one has expanded FDOSM into hexagonal FDOSM.

4. Methodology

This part gives a description of the steps required to extend FDOSM to a Hexagonal fuzzy type. This section is broken down into four sections, as shown in fig(1). Each part of this section achieves one of the research objectives.

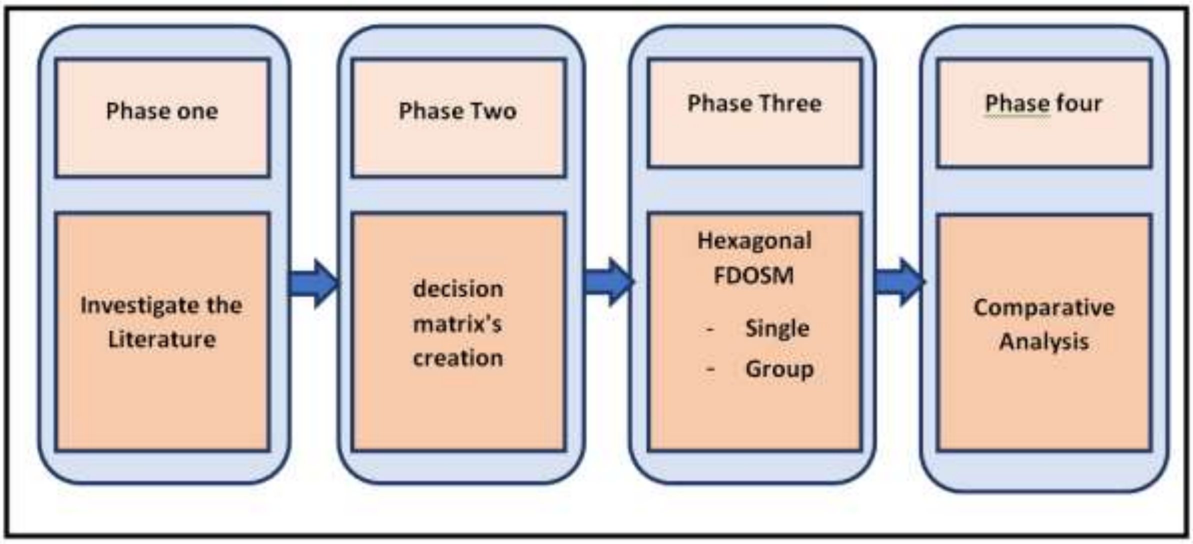

Figure 1. Methodology of extending FDOSM to a Hexagonal fuzzy type

4.1 Phase One: Investigate the Literature

In this step, all the academic literature that developed on the (FDOSM) using fuzzy numbers is studied in order to know the type of fuzzy number used in this development. We found that there is no development on the (FDOSM) using the hexagonal fuzzy number. This synopsis serves as proof for the gaps in the scholarly literature. show the main contribution of this study, as well.

4.2 Phase Two: decision matrix's creation phase

In this phase, the decision matrix is built, which includes n set of decision criteria (Cr1, ……… , Crn) and m alternatives (A1,..., Am) as follows:

$$D = \begin{matrix} A_1 \\ A_m \end{matrix} \begin{bmatrix} x_{11} & \cdots & x_{1n} \\ \vdots & \ddots & \vdots \\ x_{m1} & \cdots & x_{mn} \end{bmatrix}$$

4.3 Phase three: Hexagonal – FDOSM

This stage is divided into two main steps.

4.3.1 Data Transformation Unit

This unit converts the decision matrix(dm) into an opinion matrix(om) in two phases.

Step1: Choose the ideal solution. An ideal solution is described below:

$$A^{*} = \left\{\left[\left(\max_{i} v_{ij} \mid j \in J\right), \left(\max_{i} v_{ij} jj \in J\right), \left(Op_{ij} \in I.J\right) \mid i = 1.2.3 \ldots m\right]\right\} \quad (8)$$

where max is the optimal value for the benefit criterion, min is the optimal value for cost criteria, and Opij is the critical value where the optimal value falls between min and max .

Step2: Reference Comparison among the optimal solution and other values based upon the same criterion. Subjectively, the importance of the differences between both the optimal solution and the alternatives is evaluated. There are five scales called linguistic terms used to represent comparison,(No difference, Slight difference, Difference, Big difference and Huge difference).In the ideal solution selection process, the best solution and alternatives are compared as follows.

$$Op_{\text{Lang}} = \left\{\left(\left(\tilde{\tilde{v}} \otimes v_{ij} \mid j \in J\right) \cdot \mid i = 1.2 \ldots \ldots \ldots m\right)\right\} \quad (9)$$

where $\otimes$ denotes the comparison among the optimal solution and the alternatives. The operator $\otimes$ is utilised as the scale. This block's result is a linguistic term opinion matrix that is prepared to be converted into fuzzy numbers using fuzzy membership.

$$\text{Op} = \begin{matrix} A_1 \\ A_m \end{matrix} \begin{bmatrix} op_{11} & \cdots & op_{1n} \\ \vdots & \ddots & \vdots \\ op_{m1} & \cdots & op_{mn} \end{bmatrix} \quad (10)$$

4.3.2 Data-processing unit

This phase include three steps.

Step1: in this step, we create a fuzzy decision matrix by applying Hexagonal fuzzy membership to the opinion matrix (fuzzification process). The opinion terms are substituted by HFNs, As shown in table (1). HFNs are described by their membership function, as shown in equation (3).

Table 1 . Linguistic terms to HFN

Linguistic terms	HFN
No difference(No-diff)	(1,2,4,6,7,9)
Slight difference(Slight-diff)	(2,4,6,7,9,11)
Difference(Diff)	(4,6,7,9,11,13)
Big difference(Big-diff)	(6,7,9,11,13,15)
Huge difference(Huge-diff)	(7,9,11,13,15,16)

Step2: Use the Addition operation as shown in equation (4), to combine the value of the alternatives generated in the preceding phase.
Step3: Defuzzify the aggregation result using the method of centroid defuzzificationas follows [28]:
$$= \left(\frac{3h_1+3h_2+10h_3+10h_4+5h_5+3h_6}{34}\right) \quad (11)$$
finally the lowest value is the best option.

4.3.3 Group decision making

The purpose of group decision-making is to aggregate the decisions of several experts into a single unique decision. In the academic research on group decision-making, internal and external aggregations were identified as two prevalent forms.

Internal aggregation: The objective of this type is to aggregate the decision matrixes of different Experts into a single decision matrix. Then, decision processes are implemented to the resulting decision matrix. as shown in the following equation : [25]
I = (min. arithmetic mean. max), (12)
where I is the opinion matrix for each Expert.

External aggregation: In contrast to internal aggregation, the decision matrixes are processed independently into several decisions that aggregate into a single final decision. as shown in the following equation: [25]. External aggregation $= \oplus A^*$ (13)
where ⊕ is the arithmetic mean, and A∗ is the final result for every Expert.
In this study, we will use external aggregations .

4.4 Phase Four: Comparative Analysis

A comparison of the Hexagonal-FDOSM results and the basic FDOSM results will be presented in this section.

5. Case study

Global positioning system (GPS) is a guidance, space-based location, and scheduling system created by the US Department of Defense. It appeared In the latter half of the 1960s and the beginning of the 1970s [49]. Initially designed for military usage [50]. It is an important tool and is used daily in large numbers that may reach millions. Energy consumption is a major problem brought on by smart devices. Authors of some research papers have suggested a technique for power balancing that makes use of MCDM and a GPS that can be adjusted to suit various needs. In this case study, nine alternatives (GPS mode) were evaluated according to three criteria (i.e. power, Tno-pos and accuracy [25]. The two major possibilities in this case—static and dynamic positions—were addressed.

6. Results and Discussion

In this part, the Hexagonal FDOSM method will be applied to the GPS case study, and the results will be extracted and discussed

6.1 Decision matrix's creation phase

In this paragraph, a decision matrix is created for the GPS case study, which consists of nine alternatives and three criteria, namely Power, Tno-pos, and Accuracy . as shown in Table (2).

Table 2. GPS case study (decision matrix)

GPS receiver Operation Mode	Static Positioning.			Dynamic Positioning.		
	Pw	**Tp**	**Ac**	**Pw**	**Tp**	**Ac**
m0	1.9860	1623.80	10.30	2.060	1677.20	13.10
m1	2.2550	711.60	9.20	2.3560	870	11.340
m2	2.7730	691.80	8.80	2.970	861.20	10.960
m3	3.2960	674.40	8.10	3.5020	840.20	10.520
m4	4.0850	655.60	8	4.2060	820.40	9.740
m5	4.7650	637.60	7.90	4.9090	713.20	9.30
m6	4.8890	629.80	7.60	5.0880	690.80	8.980
m7	4.9770	620.40	7.20	5.090	682.40	8.50
m8	5.060	616.40	6.70	5.0920	677.40	8.340

Power=Pw, Tno-pos=Tp, Accuracy=Ac

6.2 Hexagonal – FDOSM

At this phase, the work is divided into three steps, which are as follows:

6.2.1 Data transformation unit

At this part, each expert selects the optimal solution and compares it with other values of the same criterion. The result of the comparison is linguistic terms. As a result of this step, the decision matrix is converted into an opinion matrix using linguistic terms as shown in tables (3), (4), (5), (6)

Table 3. opinion matrix (Expert 1)

GPS receiver Operation Mode	Static Positioning			Dynamic Positioning		
	Pw	**Tp**	**Ac**	**Pw**	**Tp**	**Ac**
m0	No-diff	Huge-diff	Big-diff	No-diff	Huge-diff	Huge-diff
m1	No-diff	Diff	Big-diff	No-diff	Diff	Big-diff
m2	Slight-diff	Slight-diff	Big-diff	Slight-diff	Slight-diff	Big-diff
m3	Slight-diff	Slight-diff	Big-diff	Slight-diff	Slight-diff	Big-diff
m4	Slight-diff	Slight-diff	Big-diff	Diff	Slight-diff	Diff
m5	Slight-diff	No-diff	Big-diff	Diff	Slight-diff	Diff
m6	Diff	No-diff	Diff	Diff	No-diff	Slight-diff
m7	Diff	No-diff	Slight-diff	Diff	No-diff	Slight-diff
m8	Big-diff	No-diff	No-diff	Diff	No-diff	No-diff

Table 4. opinion matrix (Expert 2)

GPS receiver Operation Mode	Static Positioning			Dynamic Positioning		
	Pw	**Tp**	**Ac**	**Pw**	**Tp**	**Ac**
m0	No-diff	Huge-diff	Big-diff	No-diff	Huge-diff	Diff
m1	No-diff	Big-diff	Big-diff	No-diff	Diff	Diff
m2	Slight-diff	Big-diff	Diff	Slight-diff	Diff	Diff
m3	Slight-diff	Diff	Diff	Diff	Diff	Slight-diff
m4	Diff	Diff	Diff	Diff	Slight-diff	Slight-diff
m5	Diff	Diff	Slight-diff	Big-diff	Slight-diff	Slight-diff
m6	Big-diff	Slight-diff	Slight-diff	Big-diff	Slight-diff	Slight-diff
m7	Big-diff	Slight-diff	Slight-diff	Big-diff	No-diff	Slight-diff
m8	Huge-diff	No-diff	No-diff	Big-diff	No-diff	No-diff

Table 5. opinion matrix (Expert 3)

GPS receiver Operation Mode	Static Positioning			Dynamic Positioning		
	Pw	**Tp**	**Ac**	**Pw**	**Tp**	**Ac**
m0	No-diff	Huge-diff	Diff	No-diff	Huge-diff	Diff
m1	No-diff	Diff	Diff	No-diff	Diff	Diff
m2	Slight-diff	Diff	Diff	Diff	Diff	Diff
m3	Diff	Slight-diff	Diff	Diff	Slight-diff	Diff
m4	Diff	Slight-diff	Diff	Big-diff	Slight-diff	Diff
m5	Diff	No-diff	Diff	Huge-diff	Slight-diff	Slight-diff

m6	Diff	No-diff	Diff	Huge-diff	No-diff	Slight-diff
m7	Diff	No-diff	Slight-diff	Huge-diff	No-diff	Slight-diff
m8	Big-diff	No-diff	No-diff	Huge-diff	No-diff	No-diff

Table 6. opinion matrix (Expert 4)

GPS receiver Operation Mode	Static Positioning			Dynamic Positioning		
	Pw	**Tp**	**Ac**	**Pw**	**Tp**	**Ac**
m0	No-diff	Huge-diff	Huge-diff	No-diff	Huge-diff	Huge-diff
m1	No-diff	Huge-diff	Big-diff	Slight-diff	Huge-diff	Huge-diff
m2	Slight-diff	Big-diff	Big-diff	Slight-diff	Huge-diff	Big-diff
m3	Diff	Big-diff	Big-diff	Diff	Huge-diff	Big-diff
m4	Diff	Diff	Big-diff	Diff	Big-diff	Big-diff
m5	Big-diff	Diff	Diff	Big-diff	Big-diff	Diff
m6	Huge-diff	Slight-diff	Slight-diff	Big-diff	Big-diff	Diff
m7	Huge-diff	No-diff	No-diff	Huge-diff	Diff	Slight-diff
m8	Huge-diff	No-diff	No-diff	Huge-diff	No-diff	No-diff

6.2.2Data-processing unit

This step is divided into three secondary steps

6.2.2.1 In the first step, the opinion matrix is converted into a fuzzy opinion matrix by replacing the linguistic terms with Hexagonal fuzzy numbers according to the compensation table (1), so we get a fuzzy opinion matrix.

6.2.2.2 In the second step, Using an addition equation(4), aggregate the results of the previous step for each alternative.

6.2.2.3 In the third step, the defuzzification equation is applied to the previous matrix to obtain the final result for each decision maker, as shown in Table (7).

Table 7. The end outcome of Hexa- FDOSM for all Experts(Ep).

	Ep1				Ep2				Ep3				Ep4			
Alternative	Static Positioning		Dynamic Positioning		Static Positioning		Dynamic Positioning		Static Positioning		Dynamic Positioning		Static Positioning		Dynamic Positioning	
	Sc	ra	Sc	ra	Sc	ra	Sc	ra	Sc	ra	Sc	ra	Sc	ra	Sc	ra
m0	27.3	7	29.2	6	27.3	7	25.4	7	25.4	5	25.4	8	29.2	7	29.2	5

	8		0		8		7		7		7		0		0	
m1	23.64	6	23.64	5	25.55	6	21.73	3	21.73	3	21.73	1	27.38	5	30.82	7
m2	23.55	5	23.55	4	25.26	5	23.35	5	23.35	4	25.05	5	27.17	4	29	4
m3	23.55	5	23.55	4	23.35	2	23.35	5	23.35	4	23.35	3	28.88	6	30.70	6
m4	23.55	5	23.35	3	25.05	4	21.64	2	23.35	4	25.26	6	26.97	3	28.88	3
m5	21.94	4	23.35	3	23.35	2	23.55	6	21.73	3	25.38	7	26.97	3	28.88	3
m6	21.73	3	20.02	2	23.55	3	23.55	6	21.73	3	23.76	4	25.38	2	28.88	3
m7	20.02	1	20.02	2	23.55	3	21.94	4	20.02	1	23.76	4	22.14	1	27.08	2
m8	20.32	2	18.41	1	22.14	1	20.32	1	20.32	2	22.14	2	22.14	1	22.14	1

Score=sc, rank=ra

6.2.3 Group decision-making

In this step, aggregate the decisions(final result) of several experts into a single unique decision using the type of external Group decision-making, as shown in Table (8).

Table 8. The end outcome of the group Hexa- FDOSM

	Static Positioning		Dynamic Positioning	
Alternatives	sc	Ra	Sc	Ra
m0	27.36029	9	27.33824	9
m1	24.58088	5	24.48529	4
m2	24.83824	8	25.24265	7
m3	24.78676	7	25.24265	6
m4	24.73529	6	24.78676	5
m5	23.5	4	25.29412	8
m6	23.10294	3	24.05882	3
m7	21.44118	2	23.20588	2
m8	21.23529	1	20.75735	1

Score=sc, rank=ra

FDOSM holds that the optimal alternative is the nearest to the linguistic word (no-difference) . In Table (8), the optimal alternative in (Static Positioning) is (m8), and in (Dynamic Positioning) it is (m8) since it contains the nearest value to the language word (no-difference) . the worst alternative in (Static Positioning) is (m0), and in (Dynamic Positioning) it is (m0) since it contains

the furthest value to the language word (no-difference)
The basic FDOSM was applied to the same case study(GPS) for the four decision-makers. The best option was the lowest value. Table(10) presents the end outcome of applying the fundamental FDOSM to a group.

Table 9. The final results of FDOSM for dynamic and static positions for every Expert(Ep)

	Ep1				Ep2				Ep3				Ep4			
Alternatives	Static Positioning		Dynamic Positioning		Static Positioning		Dynamic Positioning		Static Positioning		Dynamic Positioning		Static Positioning		Dynamic Positioning	
	sc	ra	sc	ra	sc	ra	sc	Ra	sc	ra	sc	ra	Sc	ra	sc	ra
m0	0.58	9	0.63	9	0.58	9	0.51	9	0.51	9	0.51	7	0.63	8	0.64	3
m1	0.46	8	0.46	8	0.52	8	0.39	4	0.38	3	0.39	2	0.57	4	0.69	8
m2	0.44	5	0.44	4	0.51	6	0.44	7	0.44	6	0.52	9	0.57	4	0.64	3
m3	0.44	5	0.44	4	0.44	4	0.44	7	0.44	6	0.44	5	0.65	9	0.71	9
m4	0.44	5	0.44	6	0.52	7	0.37	2	0.44	6	0.51	7	0.58	6	0.65	5
m5	0.38	3	0.44	6	0.44	4	0.43	5	0.38	3	0.49	6	0.58	6	0.65	5
m6	0.39	4	0.32	2	0.43	2	0.43	5	0.38	3	0.43	3	0.49	3	0.65	5
m7	0.32	1	0.32	2	0.43	2	0.38	3	0.31	1	0.43	3	0.38	1	0.57	2
m8	0.33	2	0.26	1	0.38	1	0.32	1	0.32	2	0.38	1	0.38	1	0.38	1

Score=sc, rank=ra

Table 10. The final result of the group basic FDOSM

	Static Positioning		Dynamic Positioning	
Alternatives	score	Rank	Score	rank
m0	0.5750	9	0.5722	9
m1	0.4861	5	0.4806	4
m2	0.4931	6	0.5083	7
m3	0.4944	7	0.5083	7
m4	0.4958	8	0.4944	5
m5	0.4500	4	0.5069	6
m6	0.4278	3	0.4611	3
m7	0.3639	2	0.4264	2
m8	0.3556	1	0.3389	1

6.3 Comparative Analysis

in this part, we examine the differences between the end scores obtained using the basic FDOSM and the Hexagonal -FDOSM for the same case study. We note that The final result of the group is very close between FDOSM and Hexagonal-FDOSM, but in the applications of MCDM that are subject to one expert, which is common, because most applications of MCDM are subject to one expert, we notice a big difference in the results between FDOSM and Hexagonal-FDOSM.

We will explain in detail the difference for each expert through the two tables (7) and (9).
Expert1: With regard to the first expert, we note that in the case of Dynamic Positioning, FDOSM gave the alternatives (m2,m3,m4,m5) the same Rank, despite the complete difference in their values. As for the Hexagonal-FDOSM, it was given an (m2,m3) certain Rank and (m4,m5) another rank .

Expert 2: we note that in the case of Static Positioning, FDOSM gave the alternatives (m1,m4) the same Rank, despite the complete difference in their values. while Hexagonal-FDOSM, it was give(m1) certain rank and (m4) another rank.

Expert 3: we note that in the case of Dynamic Positioning, FDOSM gave the alternatives (m0,m4) the same Rank, despite the complete difference in their values. while Hexagonal-FDOSM,it was give(m0) certain rank and (m4) another rank.

Expert 4: we note that in the case of Static Positioning, FDOSM gave the alternatives (m1,m2 the same Rank, despite the complete difference in their values. while Hexagonal-FDOSM, i t was give(m1) certain rank and (m2) another rank.

Expert 4: we note that in the case of Dynamic Positioning, FDOSM gave the alternatives (m0,m2) the same Rank, despite the complete difference in their values. while Hexagonal-FDOSM, it was give(m0) certain rank and (m2) another rank.

Finally, can see clearly the Hexagonal-FDOSM method addresses the uncertainty problem better than the traditional fuzzy number that the FDOSM method suffers from in applications of MCDM that subject to a single expert

7. Conclusions

One of the newest techniques in the MCDM strategy is FDOSM. Numerous scholars have expanded the FDOSM into another fuzzy setting and used it to tackle various MCDM issues. However, it still suffers from the problem of uncertainty, and to solve this problem, we introduced a new extension of the FDOSM with a Hexagonal Fuzzy number, namely the Hexagonal FDOSM.

Here, we presented the expansion of the FDOSM into the Hexagonal-FDOSM using the research methodology. Additionally, we used a GPS case study to apply the Hexagonal-FDOSM. In comparison to earlier developments of FDOSM, the final findings demonstrate that the

Hexagonal-FDOSM tackles the issue of uncertainty to a significant degree. in future work, researchers can use the Hexagonal-FDOSM to address issues that have already been resolved with the basic FDOSM . The FDOSM can also be expanded into other fuzzy settings, and the outcomes can be contrasted with those of the fundamental and hexagonal FDOSMs.

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