Single Input Single Output Fuzzy Logic Control Based Sliding Surface Adjustment of Second-Order Sliding Mode Controllers

Basheer V P[1], Abdul Kareem[2] and Ganesh Aithal[3]
[1] Research Scholar, Department of Electronics and Communication Engineering, Visvesvaraya Technological University, Belagavi, Karnataka, India, mailtovpb@gmail.com
[2] Department of Electronics and Communication Engineering, Moodlakatte Institute of Technology, Kundapura, India, afthabakareem@gmail.com
[3] Department of Electronics and Communication Engineering, Sri Madhwa Vadiraja Institute of Technology & Management, Udupi, India, ganeshaithal@gmail.com

Abstract:
In this research work, a novel second-order sliding mode control with a fuzzy logic based time-varying sliding surface is proposed. The time-varying sliding surfaces is an effective sliding surface design strategy for improving the controller performance. A sliding variable with relative of degree 2 is first built by accounting for the model uncertainties and external disturbances in the mathematical model. Then, to improve the tracking performance of the system under control, a time-varying sliding surface based on a straightforward single input-single output fuzzy logic inference system is proposed. The system's reaching conditions, stability, and robustness are all ensured by the proposed controller. The proposed controller lends itself well to simple design and implementation. Theoretical analysis demonstrates the global finite-time stability of the resulting closed-loop system. The proposed controller is studied in comparison with a traditional second-order sliding mode controller with a fixed sliding surface using MATLAB/SIMULINK for a nonlinear system. The results show that the proposed controller exhibits a better dynamic performance compared to the standard second-order sliding mode controller with a fixed sliding surface.

Keyword: Second-order sliding mode control, Sliding surface rot ation, Fuzzy logic control, Error convergence, Tracking accuracy
DOI: 10.24297/j.cims.2023.10.6

1. Introduction

Sliding mode control is a well-known control method that has been successfully and widely applied to dynamic uncertain systems. This popularity can be attributed to the sliding mode control's desirable properties, which include resistance to external shocks, parameter violations, and uncertainty. The sliding mode control strategy also offers a clear, user-friendly algorithm. The two steps of the sliding mode control design are the construction of the desired sliding surface and the enforcement of the sliding mode. The conventional sliding mode controller uses either relay controllers or unit controllers. One of the fundamental problems with these control systems is that switching and temporal delays in system dynamics prevent the system trajectory from reaching the ideal sliding mode, which results in chattering, a high-frequency oscillation. Additionally, while the system states are in the reaching mode, the classic sliding mode controller with a fixed sliding surface has the limitation that the tracking error cannot be easily managed, making the system vulnerable to parameter changes.

Second-order sliding mode (SOSM) control is a control technique that is based on the use of higher-order sliding modes to ensure robustness and tracking performance in the presence of uncertainties and disturbances. In SOSM control, the control law is designed to force the system state onto a sliding surface, which is a manifold of reduced dimensionality, such that the system dynamics are constrained to evolve along the surface. The performance of a second order sliding mode controller very much depends on the sliding surface. There is no formal and hard rule for the design of sliding surface. Hence, the design of optimal sliding surface is an almost impossible task. In addition, a second-order sliding mode controller with a fixed sliding surface is more susceptible to parameter changes when it is in reaching mode. This sensitivity can be decreased by shortening the duration of the reaching mode. Hence, a time-varying sliding surface is proposed in this work.

The concept behind the time-varying sliding surface is to rotate the sliding surface in the direction of improved dynamic performance. Also, SOSM control technique with time-varying sliding surface is a viable choice to achieve tracking of time-varying references or rejection of time-varying disturbances. The design of the SOSM controller with a time-varying sliding surface typically involves selecting a Lyapunov function that measures the distance between the system state and the sliding surface. The control law is then derived based on the gradient of the Lyapunov function, which drives the system towards the sliding surface. Overall, the SOSMC technique with a time-varying sliding surface can be a powerful tool for achieving robust and accurate control of uncertain systems. However, the design of the controller can be challenging and requires a good understanding of both the system dynamics and the control theory involved. Also, there is no strict and hard rules for rotation of the sliding surface in the direction of improved dynamic performance, only some approximate rules are available.

Sliding mode control is an appropriate strategy for robust control because its decreased order dynamics offer desirable benefits like matching uncertainties and disturbances and sensitivity to parameter fluctuations.[1] Sliding mode controllers have been shown to be effective for stabilizing uncertain nonlinear systems that contain nonlinearities and uncertainties [2]. A framework for the application of robust and smooth second-order sliding mode control to a class of underactuated mechanical systems for the realization of high-performance control applications in [3]. Particularly, external disturbances, parameter uncertainties, etc. affect practically all real systems inexorably, degrading the performance of many control schemes. Since un-actuated states are significantly more sensitive to disturbances than actuated ones and are therefore very challenging to stabilize, such a problem is made even worse for underactuated systems.[4]

Even though sliding mode control is an effective control technique that may create a closed-loop system that is particularly resilient to plant uncertainty and outside disturbances, it has chattering problems. A good method for reducing chattering is second-order sliding mode control. The chattering issues in the first-order SMC can be effectively reduced by the SOSM [5-9]. One technique to lessen chattering is to use a continuous compensation term to lessen the effects of the uncertainty since the chattering is proportional to the switching gain's magnitude, which is chosen to be greater than the bounded value of the uncertainty and disturbance.[10]

Even though the sliding surface can be achieved under SMC in a finite-time sense, the linear sliding variable design is only enough to cause the system to converge to the origin asymptotically rather than in a finite-time sense [11]. The SOSM method can also be suitable for the asymmetric output constraint problem [12]. SMC-based controller technique is applied to buck converter circuit in [13]. For relative-degree-2 nonlinear uncertain single-input-single-output (SISO) systems, a unique second-order sliding mode control approach is suggested in [14]. For a broad class of uncertain nonlinear systems, the globally fixed-time control problem is examined in [15].

Fuzzy logic is a popular control technique that is widely used in various industrial applications due to its ability to handle uncertainties and approximations. [16-17] It is particularly useful for control problems that are difficult to model mathematically. The problem of developing a fuzzy adaptive second-order sliding mode (SOSM) controller for a particular class of nonlinear systems has been addressed in [18]. To stop the chattering, the fuzzy sliding mode controller (FSMC) has been designed using a sliding mode control (SMC) technique combined with fuzzy logic in [19]. An innovative adaptive super-twisting sliding mode controller based on fuzzy logic is used in [20] to regulate dynamic uncertain systems.

The disadvantage of the traditional second-order sliding mode control with a fixed sliding surface is that the system performance is highly dependent on the sliding surface, even though the second-order sliding mode control scheme for the control of uncertain systems reduces the chattering phenomenon of the classical first-order sliding mode controller and guarantees higher accuracy in the presence of system imperfections and uncertainties. Finding the ideal sliding surface slope value is a difficult and time-consuming task. A super-twisting sliding mode control of a dynamic uncertain system with a time-varying sliding surface is proposed in [21]. A

SOSM algorithm based on the Lyapunov method and the saturation technique in [22]. The SOSMC technique with a time-varying sliding surface can be a powerful tool for achieving robust and accurate control of uncertain systems [23]. Using time-varying sliding surfaces rather than continuous ones is an effective sliding surface design technique for enhancing controller performance. In [24], a time-varying sliding surface adjustment using a two-input single-output fuzzy logic controller is proposed for a ship steering model.

This paper proposes a novel single-input, single-output (SISO) fuzzy logic control-based second-order sliding mode control system. The main advantage of the recommended control approach is that the sliding surface can be modified online in accordance with the values of the sliding variables to achieve the desired performance. Additionally, the sliding surface can rotate either clockwise or anticlockwise. The sliding surface change is computed utilizing a single-input single-output fuzzy logic control method, which results in a relatively simple methodology and quick computation time. Results from computer simulations show that the proposed control method outperforms the conventional second-order sliding mode controller with a fixed sliding surface.

2. Design of Proposed SOSM Controller

The proposed SOSM control technique is the modification of the SOSM control algorithm presented in [18]. The control design process has two steps. A novel SOSM controller is initially developed step-by-step in the first stage utilizing a modified version of the SOSM technique [25], and a thorough mathematical analysis is also performed. The second phase presents a comprehensive simulation strategy for the suggested SOSM algorithm.

2.1 Brief Description of SOSM

For instance, consider the nonlinear dynamical system.

$$\dot{x} = f(t,x) + g(t,x)U, s = s(t,x) \quad (1)$$

where $x \in \mathbb{R}^n$ is the system state and $U \in \mathbb{R}$ is the control input. *f(t, x)* and *g(t, x)* are the Smooth functions and the output is $s \in \mathbb{R}$ (Sliding Variable). The sliding variables s and $\dot{s}$ are taken to be known. If the sliding variable s is considered to have a relative degree of $r = 2$ in relation to the controller U, one has

$$\ddot{s} = a(t,x) + b(t,x)U \quad (2)$$

where $a(t,x) = \ddot{s}|_{U=0}$and $b(t,x) = \frac{\partial \ddot{s}}{\partial U}$ The SOSM controller has two modes: U = 1 or U = -1. It is clear from the following relation that the switch μ can be identified [26]:

$$\mu = \frac{1}{2}\left(1 + sign(U)\right) \quad (3)$$

However, it is evident from (3) that the sign function will result in an endless switching frequency whenever the sliding variables reach $[\dot{s}]^2 + \beta_1 s = 0$ This suggests that the switching frequency is too high for controller (3) to be implemented directly for the buck converter. Although it is impossible to know how large the range is, the operation frequency can still be limited within it. This can be achieved via a hysteresis modulation. In this paper, we rephrase (3) as

$$sat(\lceil\dot{s}\rfloor^2+\beta_1 s)=\begin{cases}-1, & for\lceil\dot{s}\rfloor^2+\beta_1 s<-1\\ \lceil\dot{s}\rfloor^2+\beta_1 s, & for-1<\lceil\dot{s}\rfloor^2+\beta_1 s<1\\ 1, & for\lceil\dot{s}\rfloor^2+\beta_1 s>1\end{cases} \quad (4)$$

providing the region indicated by $\Omega=-1<\lceil\dot{s}\rfloor^2+\beta_1 s<1$

The switching procedure will not take place in the region Ω after this update. As a result, this modification can be used to reduce the SOSM control's infinite switching frequency. In fact, the output voltage error will converge to the region where $|\lceil\dot{s}\rfloor^2+\beta_1 s|<1$.

It should be noted that the sliding variables will converge to the region $|\lceil\dot{s}\rfloor^2+\beta_1 s|<1$. It is simple to acquire that $\dot{s}|\dot{s}|<1-\beta_1 s$. If $V(s)=\frac{1}{2}s^2$. A quick computation provides us with $\dot{V}(s)\leq\frac{-\beta_1 s^2+|s|}{|\dot{s}|}$, this suggests that the sliding variables will eventually converge to the region $s:|s|\leq\frac{1}{\beta_1}$

2.2 SOSM with Time-Varying Sliding Surface

An issue with a second-order sliding mode controller that has a fixed sliding surface is that the system is more susceptible to parameter changes when it is in reaching mode. This sensitivity can be decreased by shortening the duration of the reaching mode. Furthermore, it is difficult and time-consuming to determine the appropriate value of the sliding surface slope. A sliding surface design strategy that effectively improves controller performance is the use of time-varying sliding surfaces as opposed to fixed ones. As a result, a key component of second-order sliding mode control systems is the method for altering the sliding surface online.

Designing an SOSM controller U is now necessary for the output x_1 to follow the desired value x_{1d}. To make the expression easier to understand, we first define $\lceil x\rfloor^\alpha=|x|^\alpha$sign (x). The SOSM controller for system (2) is designed as

$$U=-sign(\lceil\dot{s}\rfloor^2+\beta_1(s,\dot{s})s) \quad (5)$$

with an appropriately tuned value for $\beta_1(s,\dot{s})$.

The goal of this paper is to design a control strategy in which the output x_1 closely tracks a desired value x_{1d} in the presence of the lumped disturbances $w_1(t)$ and $w_2(t)$.

Three lemmas that form the cornerstone of the essential resources for the ensuing controller design are listed at the end of this section

Lemma 1 (see [21]): The following inequality exists if $p_1>0$ and $0<p_1\leq 1$:

$$|\lceil x\rfloor^{p_1p_2}-\lceil y\rfloor^{p_1p_2}|\leq 2^{1-p_2}|\lceil x\rfloor^{p_1}-\lceil y\rfloor^{p_1}|^{p_2}\forall x,y\in\mathbb{R} \quad (6)$$

Lemma 2 (see [18]): Let the constants c and d be positive. The following inequality is true for any positive function γ > 0:

$$|y|^d\leq\frac{c}{c+d}\gamma|x|^{c+d}+\frac{d}{c+d}\gamma^{-\frac{c}{d}}|y|^{c+d}\forall x,y\in\mathbb{R} \quad (7)$$

Lemma 3 (see [22]): Assume that p is a real number, where $0<p<1$: then one has

$$(|x_1|+\cdots|x_n|)^p\leq|x_1|^p+\cdots|x_n|^p,\forall x_i\in\mathbb{R},i=1,\ldots,n. \quad (8)$$

In light of the SOSM dynamics (2), controller (5), in the sense that, under controller (5), output x_1 will track the desired value x_{1d} in a limited time, allows for the finite-time formation of SOSM s = ṡ = 0.

Assume y_1 = s and y_2 = ṡ. It is possible to rewrite controller (5) and system (3) as

$$\dot{y}_1 = y_2,\ \dot{y}_2 = a(t,x) + b(t,x)U \quad (9)$$

$$U = -sign(\lceil y_2 \rfloor^2 + \beta_1(y_1, y_2)y_1) \quad (10)$$

respectively. The adding a power integrator method suggested in [27] and [28] will be used to demonstrate the finite-time stability of the closed-loop systems (9) and (10) in the sections that follow. There will be two steps in the proof. In order to stabilise y_1 to zero, a virtual controller called y_1^* will first be built. To ensure that the state y_2 will track y_1^* in finite time, the real controller U will be created

Step 1: Lyapunov function the one we select is $V_1(y_1) = \frac{2|y_1|^{\frac{5}{2}}}{5}$ The result of taking the derivative of $V_1(y_1)$ is

$$\dot{V}_1(y_1) = \lceil y_1 \rfloor^{\frac{3}{2}} y_2 = \lceil y_1 \rfloor^{\frac{3}{2}} y_2^* + \lceil y_1 \rfloor^{\frac{3}{2}}(y_2 - y_2^*) \quad (11)$$

in which y_2^* is a virtual control law. Designing y_2^* so that $y_2^* = -\beta_1(y_1, y_2)^{\frac{1}{2}} \lceil y_1 \rfloor^{\frac{1}{2}}, \beta_1(y_1, y_2) > 0$ produces

$$\begin{aligned} \dot{V}_1(y_1) &= \lceil y_1 \rfloor^{\frac{3}{2}} y_2^* + \lceil y_1 \rfloor^{\frac{3}{2}}(y_2 - y_2^*) \\ &= -\beta_1(y_1, y_2)^{\frac{1}{2}} \lceil y_1 \rfloor^{\frac{3}{2}} \lceil y_1 \rfloor^{\frac{1}{2}} + \lceil y_1 \rfloor^{\frac{3}{2}}(y_2 - y_2^*) \\ &= -\beta_1(y_1, y_2)^{\frac{1}{2}} y_1^2 + \lceil y_1 \rfloor^{\frac{3}{2}}(y_2 - y_2^*) \end{aligned} \quad (12)$$

Step 2: Select a function as

$$V_2(y_1, y_2) = V_1(y_1) + W(y_1, y_2) \quad (13)$$

with $W(y_1, y_2) = \int_{y_2^*}^{y_2} \lceil \lceil k \rfloor^2 - \lceil y_2^* \rfloor^2 \rfloor^2 dk$.

The function $V_2(y_1, y_2)$ is C^1, positive definite, and proper, which may be simply proven. The estimation from (13) is as follows:

$$\begin{aligned} \dot{V}_2\ (y_1, y_2) \le -\beta_1(y_1, y_2)^{\frac{1}{2}} y_1^2 + \lceil y_1 \rfloor^{\frac{3}{2}}(y_2 - y_2^*) + \\ \frac{\partial W(y_1, y_2)}{\partial y_1}\dot{y}_1 + \lceil \xi \rfloor^2 \dot{y}_2 \end{aligned} \quad (14)$$

with $\xi = \lceil y_2 \rfloor^2 - \lceil y_2^* \rfloor^2$. Following that, we estimate each term on the right hand side of (14).

Using Lemma 1, we can determine

$$\begin{aligned} \lceil y_1 \rfloor^{\frac{3}{2}}(y_2 - y_2^*) &\le |y_1|^{\frac{3}{2}} \left| \lceil y_2 \rfloor^{2\times\frac{1}{2}} - \lceil y_2^* \rfloor^{2\times\frac{1}{2}} \right| \\ &\le 2^{\frac{1}{2}} |y_1|^{\frac{3}{2}} |\xi|^{\frac{1}{2}} \end{aligned} \quad (15)$$

However, with Lemma 2, we have

$$2^{\frac{1}{2}} |y_1|^{\frac{3}{2}} |\xi|^{\frac{1}{2}} \le 2^{\frac{1}{2}} \times \frac{3}{4}\gamma y_1^2 + 2^{\frac{1}{2}} \times \frac{1}{4}\gamma^{-3}\xi^2 \quad (16)$$

Using $2^{\frac{1}{2}} \times \frac{3}{2}\text{y} = \frac{\beta_1(y_1, y_2)^{\frac{1}{2}}}{4}$, one get $\text{y} = \frac{\beta_1(y_1, y_2)^{\frac{1}{2}}}{3\times 2^{\frac{1}{2}}}$. The estimation stated below is valid by (15) and (16):

$$\lceil y_1 \rfloor^{\frac{3}{2}}(y_2 - y_2^*) \le \frac{\beta_1(y_1, y_2)^{\frac{1}{2}}}{4} y_1^2 + \left(\frac{3}{\beta_1(y_1, y_2)^{\frac{1}{2}}}\right)^3 \xi^2 \quad (17)$$

Given that $\frac{\partial \lceil y_2^* \rfloor^2}{\partial y_1} = -\beta_1(y_1, y_2)$, it follows from Lemma 1 that,

$$\begin{aligned} \frac{\partial W(y_1, y_2)}{\partial y_1}\dot{y}_1 &\le \left| \lceil y_2 \rfloor^{2\times\frac{1}{2}} - \lceil y_2^* \rfloor^{2\times\frac{1}{2}} \right| |\xi| \left| \frac{\partial \lceil y_2^* \rfloor^2}{\partial y_1} y_2 \right| \\ &\le 2^{\frac{1}{2}} \beta_1(y_1, y_2) |\xi|^{\frac{3}{2}} |y_2| \end{aligned} \quad (18)$$

Since, $|y_2| = |\lceil y_2 \rfloor^2|^{\frac{1}{2}} = |\xi + \lceil y_2^* \rfloor^2|^{\frac{1}{2}} \le (|\xi| + |y_2^*|^2)^{\frac{1}{2}}$, according to the lemma 3 $|y_2| \le |\xi|^{\frac{1}{2}} + |y_2^*|$. As a result, (18) can be rephrased as

$$\begin{aligned}\frac{\partial W(y_1,y_2)}{\partial y_1}\dot{y_1} &\le 2^{\frac{1}{2}}\beta_1(y_1,y_2)|\xi|^{\frac{3}{2}}\left(|\xi|^{\frac{1}{2}}+|y_2^*|\right)\\ &\le 2^{\frac{1}{2}}\beta_1(y_1,y_2)\xi^2+2^{\frac{1}{2}}\beta_1(y_1,y_2)^{\frac{3}{2}}|\xi|^{\frac{3}{2}}|y_1|^{\frac{1}{2}}\end{aligned} \tag{19}$$

Once more applying Lemma 2, one has

$$\begin{aligned}&2^{\frac{1}{2}}\beta_1(y_1,y_2)^{\frac{3}{2}}|\xi|^{\frac{3}{2}}|y_1|^{\frac{1}{2}}\\ &\le 2^{\frac{1}{2}}\beta_1(y_1,y_2)^{\frac{3}{2}}\times\frac{1}{4}\gamma y_1^2+2^{\frac{1}{2}}\beta_1(y_1,y_2)^{\frac{3}{2}}\times\frac{3}{4}\times\gamma^{-\frac{1}{3}}\xi^2\end{aligned} \tag{20}$$

If $2^{\frac{1}{2}}\beta_1(y_1,y_2)^{\frac{3}{2}}\times\frac{1}{4}\gamma=\frac{1}{2}\beta_1(y_1,y_2)^{\frac{1}{2}}$, then $\gamma=\frac{2^{1/2}}{\beta_1(y_1,y_2)}$ follows. By using (20) and a straightforward calculation, it is evident that,

$$2^{\frac{1}{2}}\beta_1(y_1,y_2)^{\frac{3}{2}}|\xi|^{\frac{3}{2}}|y_1|^{\frac{1}{2}}\le\frac{1}{2}\beta_1(y_1,y_2)^{\frac{1}{2}}y_1^2+\beta_1(y_1,y_2)^{\frac{11}{6}}\xi^2 \tag{21}$$

Putting (21) into (19) results in

$$\frac{\partial W(y_1,y_2)}{\partial y_1}\dot{y_1}\le\frac{1}{2}\beta_1(y_1,y_2)^{\frac{1}{2}}y_1^2+(2^{\frac{1}{2}}\beta_1(y_1,y_2)+\beta_1^{\frac{11}{6}})\xi^2 \tag{22}$$

From (14) it may be inferred that by combining (17) and (22)

$$\begin{aligned}\dot{V_2}(y_1,y_2)\le&\left(\frac{27}{\beta_1(y_1,y_2)^{\frac{3}{2}}}+2^{\frac{1}{2}}\beta_1(y_1,y_2)+\beta_1(y_1,y_2)^{\frac{11}{6}}\right)\xi^2\\ &-\frac{\beta_1(y_1,y_2)^{\frac{1}{2}}}{4}y_1^2\lceil\xi\rfloor^2(a(t,x)+b(t,x)U)\end{aligned} \tag{23}$$

In light of the knowledge that $\lceil y_2\rfloor^2-\lceil y_2^*\rfloor^2=\lceil y_2\rfloor^2+\beta_1(y_1,y_2)y_1=\xi$ and $b(t,x)=\frac{V_{in0}}{L_0C_0}$, putting (12) into (23) results in.

$$\begin{aligned}\dot{V_2}(y_1,y_2)\le&-\frac{\beta_1(y_1,y_2)^{1/2}}{4}y_1^2+\Big(\frac{27}{\beta_1(y_1,y_2)^{3/2}}\\ &+2^{1/2}\beta_1(y_1,y_2)+\beta_1(y_1,y_2)^{11/6}\Big)\xi^2\\ &+\xi^2|a(t,x)|-b(t,x)\lceil\xi\rfloor^2.sign(\xi)\\ \le&-\frac{\beta_1(y_1,y_2)^{1/2}}{4}y_1^2+\Big(\frac{27}{\beta_1(y_1,y_2)^{3/2}}\\ &+2^{1/2}\beta_1(y_1,y_2)+\beta_1(y_1,y_2)^{11/6}\Big)\xi^2\\ &+\xi^2|a(t,x)|-\xi^2b(t,x)\end{aligned}$$

Based on condition (9), we are aware that

$$\begin{aligned}b(t,x)>&|a(t,x)|+\frac{27}{\beta_1(y_1,y_2)^{3/2}}+2^{1/2}\beta_1(y_1,y_2)\\ &+\beta_1(y_1,y_2)^{11/6}+\frac{1}{4}\beta_1(y_1,y_2)^{1/2}\end{aligned}$$

It suggests that $\dot{V_2}(y_1,y_2)\le-\frac{\beta_1(y_1,y_2)^{1/2}}{4}(y_1^2+\xi^2)$. Due to the fact

$$\int_{y_2^*}^{y_2}\left[\lceil k\rfloor^2-\lceil y_2^*\rfloor^2\right]^2dk\le|y_2-y_2^*||\xi|^2\le 2^{\frac{1}{2}}|\xi|^{\frac{5}{2}}$$

we get

$$V_2(y_1,y_2)\le 2\left(|y_1|^{\frac{5}{2}}+|\xi|^{\frac{5}{2}}\right) \tag{24}$$

By assuming that $c=2^{-\frac{14}{5}}\beta_1(y_1,y_2)^{\frac{1}{2}},\ \alpha=\frac{4}{5}$ and applying Lemma 3 and (24), we get $\dot{V_2}(y_1,y_2)+cV_2^{\alpha}(y_1,y_2)\le 0$. Observe that $0<\alpha<1$. By using controller (10), system (9) can be globally stabilised in finite time according to the finite-time Lyapunov theory presented in [29].

2.3 Fuzzy Logic Based Sliding Surface Adjustment of Second Order Sliding Mode Controllers.

The above algorithm is a sliding mode controller with a new sliding surface $\lceil\dot{s}\rfloor^2+\beta_1 s$. However, providing a clear formula to compute the parameter β_1 is challenging. The approximate rule for designing β_1 is derived from the study of the dependence of the system response on the slope

β_1. It is found that the controller with maximum slope β_1 leads to faster error convergence, but the tracking accuracy can be degraded. If the value of β_1 is too high, it can cause large overshoot in the system states and may lead to unacceptable performance. Therefore, there is a trade-off between error convergence time and tracking time. This can be rectified by moving the sliding surface of the second order sliding mode controller, as illustrated in figure 1. Hence, the best solution is to utilize a time-varying slope, which depends on s and $\dot{s}$, ie., $\beta_1(s,\dot{s})$.

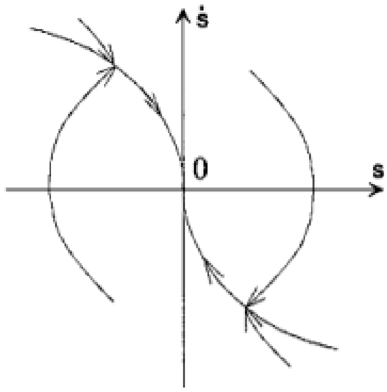

Figure. 1 Movement of Sliding Surface

In order to ensure stability, the sliding surface slope must be positive. The movement of the sliding surface can be determined by updating the value of the sliding surface slope online based on the values of the sliding variable s and its derivative $\dot{s}$. The link between the error variables and the slope of the sliding surface is not exactly modelled mathematically. Therefore, a single-input single-output fuzzy logic controller is created based on the approximation rules produced from the expert knowledge can update the sliding surface slope.

The difference between the magnitudes of s and $\dot{s}$, as provided in Equation (25) s_d is the input to the single input-single output FLC. The sliding surface slope is given by scaling the FLC output by an output scaling factor.

$$s_d = |s| - |\dot{s}| \tag{25}$$

s_d can have both positive and negative values. To guarantee stability, the FLC's output must, however, always be positive.

As shown in Figure 2, the membership functions of the input s_d are negative big (NB), negative medium (NM), negative small (NS), zero (ZE), positive small (PS), positive medium (PM), and positive big (PB), while the membership functions of the output are very very small (VVS), very small (VS), small (S), medium (M), big (B), very big (VB) and very very big (VVB) as shown in figure 3. The rule base shown in Table 1 can accomplish this.

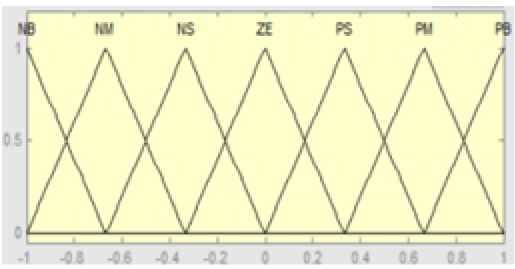

Figure. 2 Input membership functions

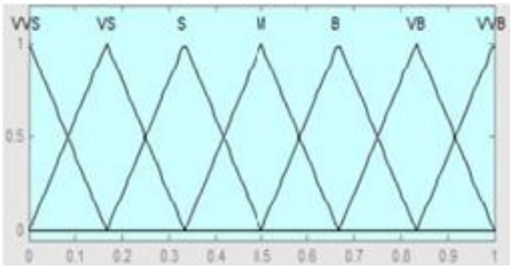

Figure. 3 Output membership functions

Table 1 One-dimensional Fuzzy Rule Base

S_d	NB	NM	NS	ZE	PS	PM	PB
Output	VVB	VB	B	M	S	VS	VVS

Defuzzification can be accomplished using the centroid approach. Figure 4 displays the input output connection of the single input, single output fuzzy logic controller.

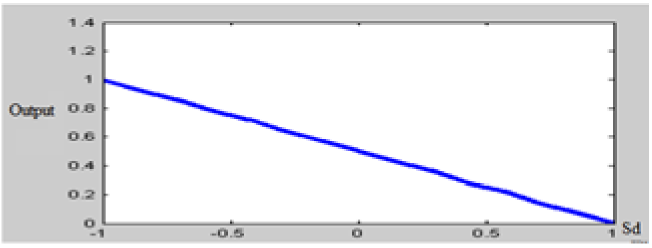

Figure 4. Input-Output Characteristics of Single Input-Single Output FLC

The control strategy can now be illustrated as seen in Figure 5.

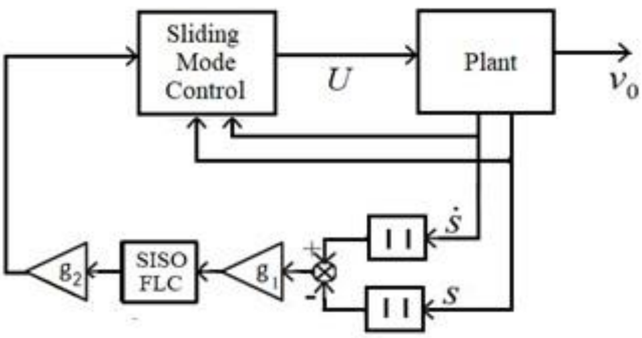

Figure 5. Proposed Control Scheme

3. Results and Discussion

The proposed controller is evaluated in comparison with a fixed sliding surface controller for a nonlinear system [18]. Figures 6-13 show the simulation results for the proposed controller and the typical second-order sliding mode controller with a fixed sliding surface. Figure 6 shows the system responses for the proposed controller and the conventional second-order sliding mode controller with different values of β_1. The proposed controller responds faster than the conventional one with a fixed sliding surface.

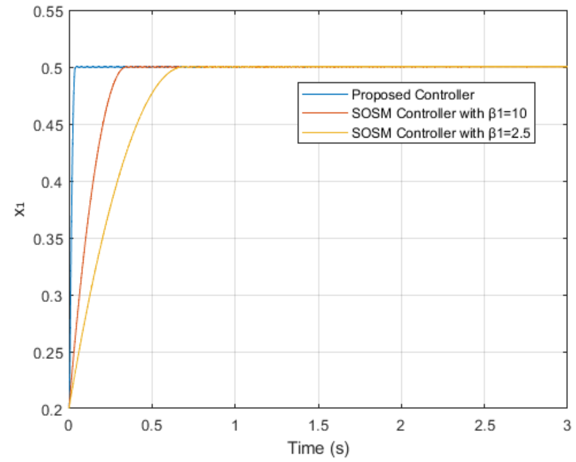

Figure 6. Responses of the Proposed Controller and SOSM Controller with $\beta_1 = 10$ and $\beta_1 = 2.5$

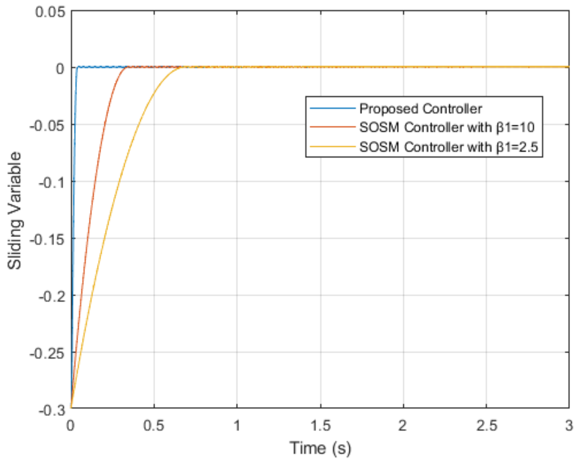

Figure 7. Sliding Variable of the Proposed Controller and SOSM Controller with $\beta_1 = 10$ and $\beta_1 = 2.5$

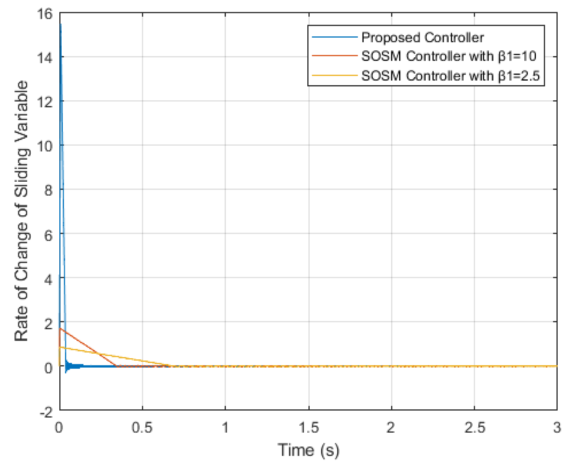

Figure 8. Rate of Change of Sliding Variable of the Proposed Controller and SOSM Controller with $\beta_1 = 10$ and $\beta_1 = 2.5$

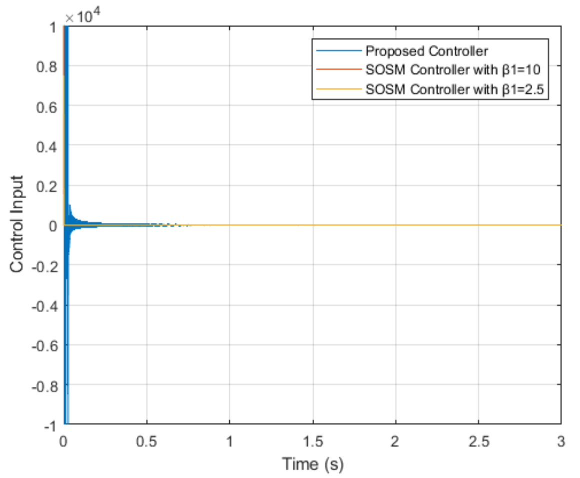

Figure 9. Control Input of the Proposed Controller and SOSM Controller with $\beta_1 = 10$ and $\beta_1 = 2.5$

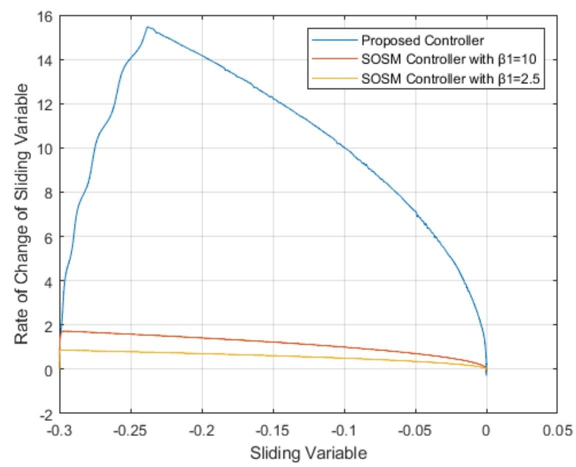

Figure 10. Error Convergence of the Proposed Controller and SOSM Controller with $\beta_1 = 10$ and $\beta_1 = 2.5$

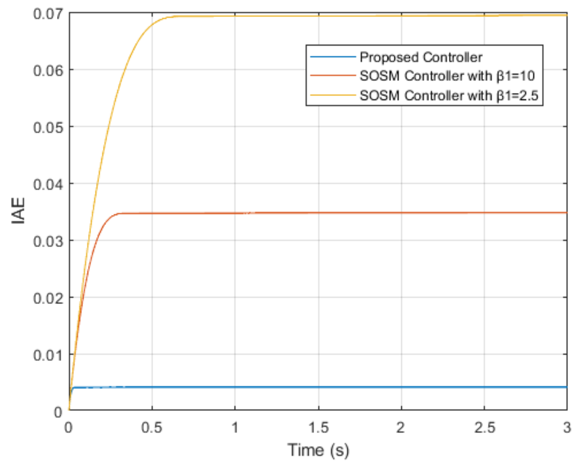

Figure 11. IAE of Sliding Variable of the Proposed Controller and SOSM Controller with $\beta_1 = 10$ and $\beta_1 = 2.5$

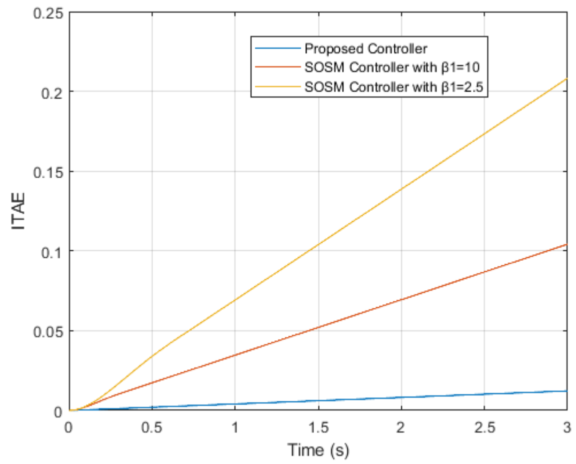

Figure 12. ITAE of Sliding Variable of the Proposed Controller and SOSM Controller with $\beta_1 = 10$ and $\beta_1 = 2.5$

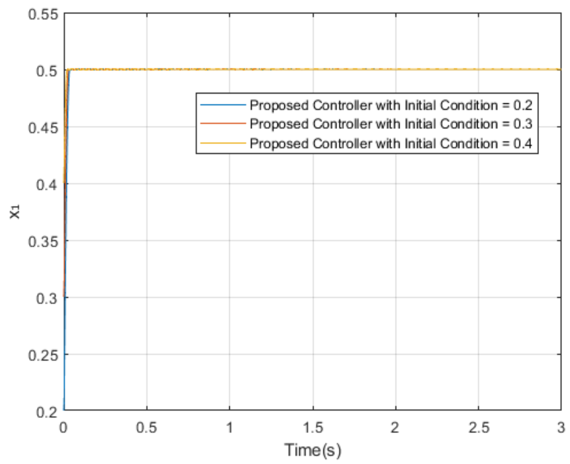

Figure 13. Responses of the Proposed Controller with Various Initial Conditions

Table 2. Performance comparison

Parameter	Proposed Controller	SOSM with β1=10	SOSM with β1=2.5
Rise Time	0.028s	0.238s	0.475s
Settling Time	0.033s	0.285s	0.565s
Peak Time	0.039s	0.347s	0.693s
Peak Overshoot	0	0	0
IAE	0.004	0.035	0.069
ITAE	0.012	0.104	0.208

The proposed controller and the typical second-order sliding mode controller with β_1=10 and β_1=2.5 have rising times of 0.028s, 0.238s, and 0.475s, and settling times of 0.033s, 0.285s, and 0.565s, respectively. In the system with the proposed controller, the time required for the response to reach the peak value is 0.039s, whereas it is 0.347s and for the typical second-order sliding mode controller β_1=10 and 0.693s for the typical second-order sliding mode controller β_1=2.5 respectively. In all cases, the steady-state error and overshoot are zero. Figure 7 depicts the sliding variables. The proposed controller achieves a faster response than the conventional second-order sliding mode controller by exerting significantly higher control effort during the first phase as shown in figure 9. Figure 10 shows how the suggested technique has a faster rate of error convergence. The proposed controller is faster throughout the response, as indicated by the IAE and ITAE curves, respectively, in Figures 11 and 12. The IAE indices for the proposed controller, the conventional second-order sliding mode controller with β_1=10, and the conventional second-order sliding mode controller with β_1=2.5 are 0.004, 0.035, and 0.069, respectively, whereas the ITAE indices are 0.012, 0.104, and 0.208, confirming the faster response of the system with the proposed controller. Figure 13 shows the responses of the proposed system with various initial conditions. The proposed method outperforms the standard method in terms of speed and robustness because it performs equally well regardless of the initial conditions. The performance metrics for the responses are summarised in Table 2.
According to simulation results, the proposed controller responds faster than a conventional second order sliding mode controller. To improve dynamic performance, stability, robustness, and tracking accuracy are not sacrificed.

4. Conclusion

This work develops a novel fuzzy logic controller-based second-order sliding mode control. It is shown that the dynamic response of the controller can be enhanced by rotating the sliding line in the phase plane using a fuzzy logic control. Results from simulations of a dynamic uncertain system are used to demonstrate the effectiveness of the suggested approach. The suggested controller is compared to a conventional second-order sliding mode controller with a fixed sliding surface using a dc-dc buck converter. Comparing the proposed controller to a second order sliding mode controller with a fixed sliding surface, the simulation results show that the suggested controller has a quick dynamic reaction, which can be read as lowering the reaching mode time and so enhancing the dynamics. In addition, the suggested control mechanism is simple, requires less computing time and easy to implement.

References

1. H. Jia, W. Shang, F. Xie, B. Zhang and S. Cong, "Second-Order Sliding-Mode-Based Synchronization Control of Cable-Driven Parallel Robots," in IEEE/ASME Transactions on Mechatronics, vol. 25, no. 1, pp. 383-394, Feb. 2020, doi: 10.1109/TMECH.2019.2960048.
2. H. Le Nhu Ngoc Thanh and S. K. Hong, "Quadcopter Robust Adaptive Second Order Sliding Mode Control Based on PID Sliding Surface," in IEEE Access, vol. 6, pp. 66850-66860, 2018, doi: 10.1109/ACCESS.2018.2877795.
3. I. Shah and F. U. Rehman, "Smooth Second Order Sliding Mode Control of a Class of Underactuated Mechanical Systems," in *IEEE Access*, vol. 6, pp. 7759-7771, 2018, doi: 10.1109/ACCESS.2018.2806568.
4. B. Lu, Y. Fang and N. Sun, "Continuous Sliding Mode Control Strategy for a Class of Nonlinear Underactuated Systems," in IEEE Transactions on Automatic Control, vol. 63, no. 10, pp. 3471-3478, Oct. 2018, doi: 10.1109/TAC.2018.2794885.
5. S. Ding, B. Zhang, K. Mei and J. H. Park, "Adaptive Fuzzy SOSM Controller Design With Output Constraints," in IEEE Transactions on Fuzzy Systems, vol. 30, no. 7, pp. 2300-2311, July 2022, doi: 10.1109/TFUZZ.2021.3079506.
6. X. He, X. Li and S. Song, "Nonsingular Terminal Sliding-Mode Control of Second-Order Systems Subject to Hybrid Disturbances," in IEEE Transactions on Circuits and Systems II: Express Briefs, vol. 69, no. 12, pp. 5019-5023, Dec. 2022, doi: 10.1109/TCSII.2022.3192030.
7. X. Shen et al., "Adaptive Second-Order Sliding Mode Control for Grid-Connected NPC Converters With Enhanced Disturbance Rejection," in IEEE Transactions on Power Electronics, vol. 37, no. 1, pp. 206-220, Jan. 2022, doi: 10.1109/TPEL.2021.3099844.
8. X. Shen et al., "High-Performance Second-Order Sliding Mode Control for NPC Converters," in IEEE Transactions on Industrial Informatics, vol. 16, no. 8, pp. 5345-5356, Aug. 2020, doi: 10.1109/TII.2019.2960550.
9. Sliding Mode Control in Power Converters and Drives: A Review Ligang Wu, Fellow, IEEE, Jianxing Liu, Senior Member, IEEE, Sergio Vazquez, Fellow, IEEE, and Sudip K. Mazumder, Fellow, IEEE

10. C. Gong, W. -K. Sou and C. -S. Lam, "Second-Order Sliding-Mode Current Controller for LC-Coupling Hybrid Active Power Filter," in IEEE Transactions on Industrial Electronics, vol. 68, no. 3, pp. 1883-1894, March 2021, doi: 10.1109/TIE.2020.2972430.
11. X. He, X. Li and S. Song, "Nonsingular Terminal Sliding-Mode Control of Second-Order Systems Subject to Hybrid Disturbances," in IEEE Transactions on Circuits and Systems II: Express Briefs, vol. 69, no. 12, pp. 5019-5023, Dec. 2022, doi: 10.1109/TCSII.2022.3192030.
12. L. Liu, S. Ding and X. Yu, "Second-Order Sliding Mode Control Design Subject to an Asymmetric Output Constraint," in IEEE Transactions on Circuits and Systems II: Express Briefs, vol. 68, no. 4, pp. 1278-1282, April 2021, doi: 10.1109/TCSII.2020.3021715.
13. Tan, S.C., Lai, Y.M., and Cheung, K.H., 2005. "A fixed-frequency pulse width modulation based quasi-sliding-mode controller for buck converters," IEEE Trans. Power Electron., vol. 20, no. 6, pp. 1379–1392
14. A. Ferrara and G. P. Incremona, "Predefined-Time Output Stabilization With Second Order Sliding Mode Generation," in IEEE Transactions on Automatic Control, vol. 66, no. 3, pp. 1445-1451, March 2021, doi: 10.1109/TAC.2020.2995667.
15. S. Shi, J. Gu, S. Xu and H. Min, "Variable-Gain Second-Order Sliding Mode Controller With Globally Fixed-Time Stability Guarantees," in IEEE Transactions on Circuits and Systems II: Express Briefs, vol. 67, no. 8, pp. 1414-1418, Aug. 2020, doi: 10.1109/TCSII.2019.2933468.
16. F. Zhang and P. Huang, "Fuzzy-Based Adaptive Super-Twisting Sliding-Mode Control for a Maneuverable Tethered Space Net Robot," in IEEE Transactions on Fuzzy Systems, vol. 29, no. 7, pp. 1739-1749, July 2021, doi: 10.1109/TFUZZ.2020.2985325.
17. D. T. Gaebele, M. E. Magaña, T. K. A. Brekken, J. C. C. Henriques, A. A. D. Carrelhas and L. M. C. Gato, "Second Order Sliding Mode Control of Oscillating Water Column Wave Energy Converters for Power Improvement," in IEEE Transactions on Sustainable Energy, vol. 12, no. 2, pp. 1151-1160, April 2021, doi: 10.1109/TSTE.2020.3035501.
18. K. Mei, S. Ding and W. X. Zheng, "Fuzzy Adaptive SOSM Based Control of a Type of Nonlinear Systems," in IEEE Transactions on Circuits and Systems II: Express Briefs, vol. 69, no. 3, pp. 1342-1346, March 2022, doi: 10.1109/TCSII.2021.3116812.
19. A. S. Huaman Loayza and C. G. Pérez Zuñiga, "Design of a Fuzzy Sliding Mode Controller for the Autonomous Path-Following of a Quadrotor," in IEEE Latin America Transactions, vol. 17, no. 06, pp. 962-971, June 2019, doi: 10.1109/TLA.2019.8896819.
20. Kareem, A., and Azeem, M.F., 2012. "A Novel Fuzzy Logic Based Adaptive Super-twisting Sliding Mode Control Algorithm for Dynamic Uncertain Systems", International Journal of Artificial Intelligence and Applications, Vol. 3, No. 4, pp. 21-34
21. Kareem, A., and Azeem, M.F., 2013. "A Novel Adaptive Super-Twisting Sliding Mode Controller with a Single Input-Single Output Fuzzy Logic Control Based Time Varying Sliding Surface", International Journal of Soft Computing, Vol. 8, No. 5, pp. 356-364.

22. K. Mei and S. Ding, "Second-Order Sliding Mode Controller Design Subject to an Upper-Triangular Structure," in IEEE Transactions on Systems, Man, and Cybernetics: Systems, vol. 51, no. 1, pp. 497-507, Jan. 2021, doi: 10.1109/TSMC.2018.2875267.
23. Kareem, A., and Azeem, M.F., 2013. "A Novel Adaptive Super-Twisting Sliding Mode Controller with a Single Input-Single Output Fuzzy Logic Control Based Moving Sliding Surface", International Journal of Control and Automation, Vol. 6, No. 3, pp. 183-197.
24. V.P, Basheer., Kareem, Abdul and Aithal, Ganesh., "Soft Computing Based Sliding Surface Adjustment of Second Order Sliding Mode Controllers: An Application to Ship Steering Model" in Journal of Survey in Fisheries Sciences, Volume 10 - Issue 1 (2023), pp. 2252-2259
25. Ding, S.,et al., 2018. "Second-Order Sliding-Mode Controller Design and Its Implementation for Buck Converters", IEEE Transactions on industrial informatics, Vol. 14, No. 5.
26. S. C. Tan, Y. M. Lai, and C. K. Tse, "Sliding Mode Control of Switching Power Converters." New York, NY, USA: CRC Press, 2012.
27. C. Qian and W. Lin, "A continuous feedback approach to global strong stabilization of nonlinear systems," IEEE Trans. Automat. Control, vol. 46, no. 7, pp. 1061–1079, Jul. 2001.
28. S. H. Ding, A. Levant, and S. H. Li, "Simple homogeneous sliding-mode controller," Automatica, vol. 67, pp. 22–32, 2016
29. S. P. Bhat and D. S. Bernstein, "Finite-time stability of continuous autonomous system," SIAM J. Control Optim., vol. 38, no. 3, pp. 751–766, 2000