Estimation Procedure of Mean under Measurement Errors in Systematic Sampling

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Abstract:

This article suggest estimation method of finite population mean for systematic sampling under measurement error and presented a class of estimators under measurements error using systematic sampling scheme. The study variable and auxiliary variables are commingled with measurement error. The properties of the estimators is achieved. The simulation study is accompanied to shows the influence of measurement error at various phase of correlation coefficient and measurements error variance.

Keywords: Proposed Estimator, Systematic Sampling, Measurement Error, Ratio, Product, Difference Estimator, Mean Square Error

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1. Introduction

Systematic sampling is widely used and more convenient than simple random sampling as it is simplest sampling scheme. Apart from simplicity, systematic sampling give precise estimate than simple random sampling and stratified random sampling under certain conditions. The use of auxiliary information is fruitful and give precise estimates. The ratio, product and difference estimator are well known consistent, biased and reliable estimates than those based on simple averages (Cochran, 1963). Swain (1964) and Shukla (1971) proposed ratio and product estimators respectively in context of systematic sampling.

Cochran (1968) and Murthy (1967) discuss the real life problem where the data is found with error. Measurement error is the difference between observed and accurate values of the variables. The measurement error first encountered by Shalabh (1997) in sampling technique. In survey sampling, measurement error is further studied by Singh and Manisha (2001), Allen and Singh (2003), Sahoo et al. (2006), Gregorre and Salas (2009). Shalabh (2017) studied correlated measurement error in perspective of ratio and product method. Singh et al. (2019) commingled error on ratio, product and mean estimator. Singh and Vishwakarma (2019) calculated the consequence of measurement error and non-response on mean estimation simultaneously.

In study related to parametric estimation in systematic sampling, But during survey sampling context to systematic sampling, observations are commingled with error. In this article, we proposed a class of estimators which contain ratio, product and difference estimator and unbiased mean estimator under measurement errors.

Suppose the units $u=(u_1,u_2,...,u_N)$ are a finite population N. The population size is distributed into k interval such that $N=nk$. To choose a sample, first unit is chosen randomly from the first k units and then select every subsequent k^{th} unit. This methods of sampling is to select a cluster randomly among k cluster (each cluster contain n units), in a way that i^{th} cluster encompasses serial numbered units $i, i+k, i+2k,...,i+(n-1)k$. Under the situation when data are observed with error. Let (x_{ij}, y_{ij}) are observed value and (X_{ij}, Y_{ij}) is true values for each i^{th} $(i=1,2,...,n)$ unit. It can be expressed in additive form as, $x_{ij}=X_{ij}+V_{ij}$ and $y_{ij}=Y_{ij}+U_{ij}$. For measurement error, the expected value of s_x^2 and s_y^2 is

$$E(s_x^2)=\sigma_{Xsy}^2+\sigma_{Vsy}^2 \text{ and } E(s_y^2)=\sigma_{Ysy}^2+\sigma_{Usy}^2$$

where σ_{Usy}^2, σ_{Vsy}^2 are variance of U and V respectively.

The systematic sample means are

$$\mu_{Ysy}=\frac{1}{nk}\sum_{i=1}^{k}\sum_{j=1}^{n}y_{ij} \quad \text{and} \quad \mu_{X sy}=\frac{1}{nk}\sum_{i=1}^{k}\sum_{j=1}^{n}x_{ij}.$$

The sample means are unbiased estimators of population means μ_{Ysy} and $\mu_{X sy}$ respectively.

$$\bar{y}_{sy}=\frac{1}{n}\sum_{j=1}^{n}y_{ij},(i=1,2,...,k) \quad ....(1)$$

$$\bar{x}_{sy}=\frac{1}{n}\sum_{j=1}^{n}x_{ij},(i=1,2,...,k) \quad .... (2)$$

To determine the bias and variance, means are expressed as the error terms e_0 and e_1, which are defined as

$$\bar{y}_{sy}=\mu_{Ysy}(1+e_0) \quad \text{and} \quad \bar{x}_{sy}=\mu_{Xsy}(1+e_1)$$

We can write

$$E(e_0)=E(e_1)=0$$

and $E(e_1^2)=\frac{1}{\mu_{Xsy}^2}\{\sigma_{Xsy}^2+\sigma_{Vsy}^2\}$, $E(e_0^2)=\frac{1}{\mu_{Ysy}^2}\{\sigma_{Ysy}^2+\sigma_{Usy}^2\}$.

$$E(e_0e_1)=\frac{\rho\sigma_{Xsy}\sigma_{Ysy}}{\mu_{Ysy}\mu_{Xsy}}, \qquad R=\frac{\mu_{Ysy}}{\mu_{Xsy}}.$$

$\sigma^2_{Ysy}=\frac{1}{k}\sum_{i=1}^{n}\left(\bar{y}_{sy}-\mu_{Ysy}\right)^2$, $\sigma^2_{Xsy}=\frac{1}{k}\sum_{i=1}^{n}\left(\bar{x}_{sy}-\mu_{Xsy}\right)^2$.

$\sigma^2_{Usy}=\frac{1}{k}\sum_{i=1}^{k}(\bar{U}_{i.})^2$ $\sigma^2_{Vsy}=\frac{1}{k}\sum_{i=1}^{k}(\bar{V}_{i.})^2$

2. Existing Estimator

The variance of systematic sample mean

$$\sigma^2_{Ysy}=\frac{1}{k}\sum_{i=1}^{n}\left(\bar{Y}_{sy}-\mu_{Ysy}\right)^2. \quad ....(3)$$

Considering that the presence of measurement error in observations, then the variance in the is obtained as

$$V\left(\bar{y}_{sym}\right)=\sigma^2_{Ysy}+\sigma^2_{Usy}. \quad ....(4)$$

$$\sigma^2_{Usy}=\frac{1}{k}\sum_{i=1}^{k}(\bar{U}_{i.})^2 \quad ....(5)$$

The usual ratio estimators Swain (1964) under the systematic random sampling is

$$\bar{y}_{Rsy}=\bar{Y}_{sy}\frac{\mu_{Xsy}}{\bar{X}_{sy}}. \quad ....(6)$$

The mean square error of the estimator is given as

$$MSE\left(\bar{y}^{*}_{Rsy}\right)=\left[\sigma^2_{Ysy}+R^2\sigma^2_{Xsy}-2R\left(\rho\sigma_{Xsy}\sigma_{Ysy}\right)\right]. \quad ....(7)$$

The situations of measurement error when the variables are recorded with error. Ratio estimators, for this scenario is defined as

$$\bar{y}_{Rsym}=\bar{y}_{sy}\frac{\mu_{Xsy}}{\bar{x}_{sy}} \quad ....(8)$$

To attain the bias and mean square error equation (8) can be expressed as

$$\bar{y}_{Rsym}=\mu_{Ysy}\left(1+e_0\right)\frac{\mu_{Xsy}}{\mu_{Xsy}\left(1+e_1\right)} \quad ....(9)$$

$$\bar{y}_{Rsym}=\mu_{Ysy}\left(1+e_0\right)\left(1+e_1\right)^{-1} \quad ....(10)$$

For bias of the estimators we obtained

$$\bar{y}_{Rsym}=\mu_{Ysy}\left(e_1^2-e_0e_1\right) \quad ....(11)$$

Taking expectation we get the bias of the estimators

$$bias\left(\bar{y}_{Rsym}\right)=\mu_{Ysy}\left(e_1^2-e_0e_1\right) \quad ....(12)$$

For mean square error it can written

$$\bar{y}_{Rsym}=\mu_{Ysy}\left(e_0-e_1\right)^{-1} \quad ....(13)$$

$$\left(\bar{y}_{Rsym}-\mu_{Ysy}\right)^2=\mu^2_{Ysy}\left(e_0^2+e_1^2-2e_0e_1\right) \quad ....(14)$$

The mean square error can be obtained as by taking expectation of (14)

$MSE\left(\overline{y^*}_{Rsym}\right) = [\sigma^2_{Ysy} + \sigma^2_{Usy} + R^2(\sigma^2_{Xsy} + \sigma^2_{Vsy}) - 2R(\rho\sigma_{Xsy}\sigma_{Ysy})]$ ....(15)

Under no commingled error, the results can be obtained by putting σ^2_{Usy} and σ^2_{Vsy} zero. This will give same results as obtained by Swain (1964). From (7) and (15) it is inferred that MSE in the presence of measurement error is always high.

The product estimators Shukla (1971) under systematic random sampling is defined as

$$\bar{y}_{Psy} = \bar{Y}_{sy} \frac{\bar{X}_{sy}}{\mu_{Xsy}} \qquad ....(16)$$

The mean square error of the estimator is

$$MSE(\bar{y}_{Rsym}) = [\sigma^2_{Ysy} - R^2\sigma^2_{Xsy} - 2\rho R\sigma_{Ysy}\sigma_{Xsy}] \qquad ....(17)$$

The situations when the variables are commingled with measurement error. Thus the product estimator is defined as

$$\bar{y}_{Psym} = \bar{y}_{sy} \frac{\bar{x}_{sy}}{\mu_{Xsy}} \qquad ....(18)$$

In order to obtain the bias and mean square error, (18) is expressed in terms of e

$$\bar{y}_{Psym} = \mu_{Ysy}(1+e_0)(1+e_1) \qquad ....(19)$$

For the bias by taking expectation we get

$$bias(\bar{y}_{Psym}) = \left(\frac{\rho\sigma_{Ysy}\sigma_{Xsy}}{\mu_{Xsy}}\right) \qquad ....(20)$$

To derive the mean square error we can write from (19) as

$$\left(\bar{y}_{Psym} - \mu_{Ysy}\right)^2 = \mu^2_{Ysy}\left(e_0^2 + e_1^2 + 2e_0e_1\right) \qquad ....(21)$$

$$MSE(\bar{y}_{Psym}) = (\sigma^2_{Ysy} + \sigma^2_{Usy} + R^2(\sigma^2_{Xsy} + \sigma^2_{Vsy}) + 2\rho R\sigma_{Ysy}\sigma_{Xsy}). \qquad ....(22)$$

By substituting the value σ^2_{Usy} and σ^2_{Vsy} equal to zero we can obtain the MSE without measurement error which is similar to Shukla (1971). From (17) and (22), it can be inferred that MSE is larger in the presence of measurement error.

The difference estimator under systematic sampling is defined as

$$\bar{y}_{dsy} = \bar{Y}_{sy} + b\left(\mu_{Xsy} - \bar{X}_{sy}\right) \qquad (23)$$

$$V(\bar{y}_{dsym}) = \sigma^2_{Ysy}(\rho^2 - 1) \qquad ... \qquad ..(24)$$

We consider the situations when the both variables are recorded with measurement error. Under that conditions, we define the difference type estimator as

$$\bar{y}_{dsym} = \bar{y}_{sy} + b\left(\mu_{Xsy} - \bar{x}_{sy}\right) \qquad ....(25)$$

In order to derive, bias and mean square error, we can write (25) as

$$\bar{y}_{dsym} = \mu_{Ysy}(1+e_0) + b\left(\mu_{Xsy} - \mu_{Xsy}(1+e_1)\right) \qquad ....(26)$$

$$\bar{y}_{dsym} = \mu_{Ysy}e_0 - be_1\mu_{Xsy} \qquad ....(27)$$

$$V\left(\bar{y}_{dsym}\right)=E\left[\mu_{Ysy}^{2}e_{0}^{2}+b^{2}e_{1}^{2}\mu_{Xsy}^{2}-2b\mu_{Ysy}\mu_{Xsy}e_{0}e_{1}\right] \quad ....(28)$$

$$\mathrm{V}(\bar{y}_{dsym})=[(\sigma_{Ysy}^{2}+\sigma_{Usy}^{2})+b^{2}(\sigma_{Xsy}^{2}+\sigma_{Vsy}^{2})-2b\rho\sigma_{Ysy}\sigma_{Xsy}] \quad ....(29)$$

The minimum variance is obtained by differentiate (29) with respect to b and equate the results zero, we attain

$$b=\frac{\rho\sigma_{Ysy}\sigma_{Xsy}}{\left(\sigma_{Xsy}^{2}+\sigma_{Vsy}^{2}\right)} \quad ....(30)$$

After putting the value of b in (30), we get minimum variance of the estimator as

$$V(\bar{y}_{dsym})=\left[(\sigma_{Ysy}^{2}+\sigma_{Usy}^{2})-\frac{\rho^{2}\sigma_{Ysy}^{2}\sigma_{Xsy}^{2}}{(\sigma_{Xsy}^{2}+\sigma_{Ysy}^{2})}\right] \quad ...(31)$$

From (24) and (31), it is concluded that MSE in the presence of measurement error is always high. After substituting the values of σ_{Usy}^{2} and σ_{Vsy}^{2} equal to zero, we can attain the MSE under no measurement error, similar as given in (24).

1. The Proposed Estimator

Considering the impact of measurement errors on the mean square error, we proposed a class of ratio, product, difference and mean estimators in the presence of measurement errors as

$$\hat{\bar{Y}}_{SM}=\bar{y}_{sy}\left(\frac{\bar{x}_{sy}}{\mu_{Xsy}}\right)^{\theta} \quad ....(32)$$

(i) When $\theta=0$, $\hat{\bar{Y}}_{SM}=\bar{y}_{sym}$ (estimator)

(ii) When $\theta=-1$, $\hat{\bar{Y}}_{SM}=\hat{\bar{Y}}_{SR}$ (ratio estimator)

(iii) When $\theta=1$, $\hat{\bar{Y}}_{SM}=\hat{\bar{Y}}_{SP}$ (product estimator).

Writing the equation (32) in terms of e, we have

$$\hat{\bar{Y}}_{SM}=\mu_{Ysy}(1+e_{0})\left(\frac{\mu_{Xsy}(1+e_{1})}{\mu_{Xsy}}\right)^{\theta} \quad ....(33)$$

$$\hat{\bar{Y}}_{SM}-\mu_{Ysy}=\mu_{Ysy}e_{0}\left[(1+e_{1})^{\theta}\right] \quad ....(34)$$

As $|e_{1}|<1$, thus $\{1+e_{1}\}^{\theta}$ is a powers a series in terms of θ. Simplifying and retaining θ up-to the second degree, we can get

$$\left[\hat{\bar{Y}}_{SM}-\mu_{Ysy}\right]=\mu_{Ysy}\left(\theta e_{0}e_{1}-\frac{\theta(1-\theta)}{2}e_{1}^{2}\right) \quad ....(35)$$

S $E(e_{0}^{2})$, $E(e_{0}e_{1})$ we obtain the bias of $\hat{\bar{Y}}_{SM}$ upto first order

$$B\left(\hat{\bar{Y}}_{SM}\right)=\mu_{Ysy}\frac{1}{k}\left[\theta\rho C_{Xsy}C_{Ysy}-\frac{\theta(\theta-1)}{2}C_{Xsy}^{2}\left\{1+\frac{\sigma_{Vsy}^{2}}{\sigma_{Xsy}^{2}}\right\}\right] \quad ....(36)$$

$$B\left(\hat{\hat{Y}}_{SM}\right)=\mu_{Ysy}\frac{1}{k}\left[\theta\rho C_{Xsy}C_{Ysy}-\frac{\theta(\theta-1)}{2}C_{Xsy}^{2}-\frac{\theta(\theta-1)}{2}C_{Xsy}^{2}\frac{\sigma_{Vsy}^{2}}{\sigma_{Xsy}^{2}}\right] \quad ....(37)$$

On squaring equation (34) and again retaining terms of e to the second degree,

$$\left(\hat{\hat{Y}}_{SM}-\mu_{Ysy}\right)^{2}=\mu_{Ysy}^{2}\left[e_0^2+\theta^2e_1^2+2\theta e_0e_1\right] \quad ....(38)$$

The *MSE* of $\hat{\hat{Y}}_{SM}$ to the first degree of approximation as obtained as

$$M(\hat{\hat{Y}}_{SM})=\mu_{Ysy}^{2}\left[C_{Ysy}^{2}\left(1+\frac{\sigma_{Usy}^{2}}{\sigma_{Ysy}^{2}}\right)+\theta^{2}C_{Xsy}^{2}\left(1+\frac{\sigma_{Vsy}^{2}}{\sigma_{Xsy}^{2}}\right)+2\theta\rho C_{Xsy}C_{Ysy}\right] \quad ....(39)$$

Differentiating partially (39) with respect to θ and equate to zero, we get the optimum value of θ as

$$\theta=\frac{-\rho C_{Xsy}C_{Ysy}}{C_{Xsy}^{2}+\frac{\mu_{Ysy}^{2}}{\mu_{Xsy}^{2}}\sigma_{Vsy}^{2}} \quad ....(40)$$

The second order derivative with respect to θ is positive thus substituting the optimum value of θ in (40), the minimum mean square error of the proposed class of estimators $\hat{\hat{Y}}_{SM}$ as

$$\min.MSE(\sigma_{Ysy}^{2})=\left(\sigma_{Ysy}^{2}+\sigma_{Usy}^{2}\right)-\frac{\rho^{2}\sigma_{Ysy}^{2}\sigma_{Xsy}^{2}}{(\sigma_{Xsy}^{2}+\sigma_{Vsy}^{2})} \quad ....(41)$$

3. Efficiency Comparison

From (41) and (31), one can conclude that, the proposed class of estimators is as efficient as difference estimators.

For $\theta=0$, the proposed class of estimators $\hat{\hat{Y}}_{SM}$ will convert into unbiased mean estimator under measurement error as $\hat{\hat{Y}}_{SM}=\bar{y}_{sy}$.

The variance of this estimator is derived from (39) by substituting the value of θ equals to zero

$$V\left(\hat{\hat{Y}}_{SM}\right)=\sigma_{Ysy}^{2}+\sigma_{Usy}^{2} \quad ....(42)$$

From (41) and (44) we can write that proposed estimator is more efficient if

$$\sigma_{Ysy}^{2}[\rho^{2}\eta_{X}]>0. \quad ....(43)$$

When $\theta=-1$, the proposed class of estimators $\hat{\hat{Y}}_{SM}$ will transform into ratio estimator under measurement error as $\hat{\hat{Y}}_{SM}=\frac{\bar{y}_{sy}}{\bar{x}_{sy}}\mu_{Xsy}=\hat{\hat{Y}}_{SR}$.

After putting the value of θ in (39) we get the mean square error of the estimator $\hat{\hat{Y}}_{SR}$ as

$$M(\hat{\hat{Y}}_{SR})=\sigma_{Ysy}^{2}+\sigma_{Usy}^{2}+R^{2}\left(\sigma_{Xsy}^{2}+\sigma_{Vsy}^{2}\right)-2\rho R\sigma_{Ysy}\sigma_{Xsy}). \quad ....(44)$$

From (41) and (44), it is revealed that $\min .M(\hat{\hat{Y}}_{SM}) \leq M(\hat{\hat{Y}}_{SR})$ if

$$\sigma_{Ysy}^2[-\rho^2\eta_{Xsy}] < \left[\frac{\mu_{Ysy}^2}{\mu_{Xsy}^2} - \frac{2\rho\sigma_{Xsy}\sigma_{Ysy}\mu_{Ysy}}{\mu_{Xsy}} + \frac{\mu_{Ysy}^2}{\mu_{Xsy}^2}\sigma_{vsy}^2\right]$$

....(45)

For $\theta = 1$, the proposed class of estimators $\hat{\hat{Y}}_{SM}$ will be product estimator under measurement error as $\hat{\hat{Y}}_{SM} = \frac{\bar{y}_{sy}}{\mu_{Xsy}}\bar{x}_{sy} = \hat{\hat{Y}}_{SP}$.

For the value of θ equal to zero in (39), the mean square error of $\hat{\hat{Y}}_{SP}$ as

$$M\left(\hat{\hat{Y}}_{SP}\right) = (\sigma_{Ysy}^2 + \sigma_{Usy}^2 + R^2\left(\sigma_{Xsy}^2 + \sigma_{Vsy}^2\right) + 2\rho R\sigma_{Ysy}\sigma_{Xsy}).$$

. ....(46)

From (41) and (46), it is revealed that $\min .M(\hat{\hat{Y}}_{SM}) \leq M(\hat{\hat{Y}}_{SP})$ if

$$\sigma_{Ysy}^2[-\rho^2\eta_{Xsy}] < \left[\frac{\mu_{Ysy}^2}{\mu_{Xsy}^2} + \frac{2\rho\sigma_{Xsy}\sigma_{Ysy}\mu_{Ysy}}{\mu_{Xsy}} + \frac{\mu_{Ysy}^2}{\mu_{Xsy}^2}\sigma_{vsy}^2\right]$$

. ....(47)

Thus from (41), (43), (45) and (47) we have

$$M(\hat{\hat{Y}}_{SM})_{Opt} \leq M\left(\hat{\hat{Y}}_{SR}\right) \leq V\left(\bar{y}_{sym}\right). \quad ....(48)$$

$$M(\hat{\hat{Y}}_{SM})_{Opt} \leq M\left(\hat{\hat{Y}}_{SP}\right) \leq V\left(\bar{y}_{sym}\right). \quad ....(49)$$

Equation (48) and (49) provide that the proposed class of estimators $\hat{\hat{Y}}_{SM}$ is better than $\hat{\hat{Y}}_{SR}$, $\hat{\hat{Y}}_{SP}$ and $\bar{y}_{sym}$ at its optimum conditions.

4. Simulation Study

Simulation study is carried to validate the results of study. The data matrix have been generated using multivariate normal distribution on X, Y, u and v for four variable with mean vector $\begin{pmatrix}\mu_{Ysy} & \mu_{Xsy} & 0 & 0\end{pmatrix}$ and covariance matrix

$$\begin{pmatrix} \sigma_{Ysy}^2 & \rho\sigma_{Xsy}\sigma_{Ysy} & 0 & 0 \\ \rho\sigma_{Xsy}\sigma_{Ysy} & \sigma_{Xsy}^2 & 0 & 0 \\ 0 & 0 & \sigma_{Usy}^2 & 0 \\ 0 & 0 & 0 & \sigma_{Vsyy}^2 \end{pmatrix}$$

The data for N=1000 has been generated by using bellow groups of parameters

μ_{Xsy} =16, μ_{Ysy} =18, σ_{Xsy}^2 = (40, 20), σ_{Ysy}^2 = (50, 30),

ρ = (-0.9,-0.5,-0.1, 0.9, 0.5, 0.1), σ_{Usy}^2 = (0, 4), σ_{Vsy}^2 = (0, 4).

Two sets of sample n=250 (k = 4) and n =100 (k = 10) has been selected under systematic sampling scheme. The mean square errors and percentage relative efficiency are achieved for all the proposed estimators. Eight numbers of tables are made for four stage of measurements errors and two stage of variance of study and auxiliary variable in estimating practise of mean.

The outcomes of simulation study are given in tables. Table 1 provide the MSE and PRE of proposed estimator and members of estimators for the various phase of correlation coefficient $(\rho = -0.9, -0.5, -0.1, 0.1, 0.5, 0.9)$. From Table 1, we conclude that proposed estimator has higher efficacy than ratio, product and mean estimator for the various phase of correlation coefficient under no commingled error. Table 2, reflect the commingled of error in MSE and PRE for ($\sigma^2_{Usy} = 4$, σ^2_{Vsy} =4) for proposed and members of estimators. We can also conclude from Table 2, that MSE is always high in the presence of measurement error. From Table 2,3,4,6,7 and Table 8, we find that MSE has higher value with increase in measurement error in study and auxiliary variable. Properties of the estimators related to correlation coefficient, MSE is always minimum for proposed estimator. For positive correlation ratio estimator is more efficient for and negative correlation product estimators is respectively. From Table 1 to 8, we can conclude that with increase in sample size MSE is increasing. For increasing value of sample size MSE is more precise and PRE is more efficient for large samples.

5. Conclusion

As we can see from simulation study of the Tables, that MSE is minimum for proposed class of estimator. Also the proposed estimator is most precise than ratio and product estimator for all correlation coefficient. It is inclusive that MSE has been constantly higher after study and auxiliary variables are observed with measurement error. MSE is high, when the degree of error is high. Measurements error effects the MSE as well as PRE of the estimator when the degree of error is high, but behavior of estimators do not alter in the occurrence of measurement error. Since PRE is the ratio, it does not precisely illustrate the consequence of measurement error. Inference based on the data that commingled with error will deceive the results. It can also be conclude that, for large values of the sample, the MSE is more precise and more proficient. Thus proposed class of estimators is useful to evaluate the population mean for commingled error.

Table 1: MSE and PRE of estimators of $\bar{Y}$ for $\rho = (-0.9, -0.5, -0.1, 0.1, 0.5, 0.9)$ when $(\sigma^2_{Ysy}, \sigma^2_{Xsy} = 50, 40)$, $(\sigma^2_{Usy}, \sigma^2_{Vsy} = 0, 0)$ and $(k = 4, 10)$

σ^2_{Usy} σ^2_{Vsy} ρ	Estimator	$k = 4$	$k = 10$

		MSE	PRE	*MSE*	PRE
0 0 -0.9	$\hat{\bar{Y}}_{SM}$	0.1516	527.7704	0.9391	521.51
	$\bar{y}_{Rsym}$	3.0406	26.31389	18.5340	26.42441
	$\bar{y}_{Psym}$	0.2002	399.6503	1.1058	442.892
	$\bar{y}_{sym}$	0.8001	100	4.8975	100
0 0 - 0.5	$\hat{\bar{Y}}_{SM}$	0.6009	133.3833	3.6373	133.3352
	$\bar{y}_{Rsym}$	2.2915	34.97709	14.5312	33.37508
	$\bar{y}_{Psym}$	0.8852	90.54451	5.1093	94.92103
	$\bar{y}_{sym}$	0.8015	100	4.8498	100
0 0 -0.1	$\hat{\bar{Y}}_{SM}$	0.8021	102.107	4.8763	101.011
	$\bar{y}_{Rsym}$	1.7628	46.46018	10.9054	45.16661
	$\bar{y}_{Psym}$	1.4854	55.13666	9.0182	54.61844
	$\bar{y}_{sym}$	0.8190	100	4.9256	100
0 0 0 .1	$\hat{\bar{Y}}_{SM}$	0.7970	100.1757	4.8505	101.0102
	$\bar{y}_{Rsym}$	1.4669	54.4277	8.9441	54.77913
	$\bar{y}_{Psym}$	1.7485	45.662	10.8066	45.33803
	$\bar{y}_{sym}$	0.7984	100	4.8995	100
0 0 0.5	$\hat{\bar{Y}}_{SM}$	0.5910	133.2318	3.6950	133.3342
	$\bar{y}_{Rsym}$	0.8908	88.39246	5.2067	94.62231
	$\bar{y}_{Psym}$	2.3005	34.22734	14.7890	33.31327
	$\bar{y}_{sym}$	0.7874	100	4.9267	100
0 0 0.9	$\hat{\bar{Y}}_{SM}$	0.1462	527.0178	0.9506	526.3833
	$\bar{y}_{Rsym}$	0.2005	384.2893	1.0957	456.6761
	$\bar{y}_{Psym}$	2.9104	26.47402	18.9404	26.41866
	$\bar{y}_{sym}$	0.7705	100	5.0038	100

Table 2: MSE and PRE of estimators of $\bar{Y}$ for $\rho = (-0.9, -0.5, -0.1, 0.1, 0.5, 0.9)$ when $(\sigma^2_{Ysy}, \sigma^2_{Xsy} = 50, 40)$, $(\sigma^2_{Usy}, \sigma^2_{Vsy} = 4, 4)$ and $(k = 4, 10)$

σ^2_{Usy} σ^2_{Vsy} ρ	Estimator	$k = 4$		$k = 10$	
4 4 -0.9		*MSE*	PRE	*MSE*	PRE

	$\hat{\hat{Y}}_{SM}$	0.2697	295.50	1.6541	306.08
	$\bar{y}_{Rsym}$	3.0993	28.09	19.1890	27.36
	$\bar{y}_{Psym}$	0.3515	232.80	1.9683	261.47
	$\bar{y}_{sym}$	0.8393	100.00	5.1811	100.00
4 4 - 0.5	$\hat{\hat{Y}}_{SM}$	0.6852	123.32	4.2263	125.66
	$\bar{y}_{Rsym}$	2.4339	36.11	15.6814	34.51
	$\bar{y}_{Psym}$	1.0296	82.89	6.1171	87.61
	$\bar{y}_{sym}$	0.8576	100.00	5.3347	100.00
4 4 -0.1	$\hat{\hat{Y}}_{SM}$	0.8449	100.76	5.1768	100.82
	$\bar{y}_{Rsym}$	1.8516	45.96	11.6698	44.94
	$\bar{y}_{Psym}$	1.5834	53.54	9.8178	53.37
	$\bar{y}_{sym}$	0.8516	100.00	5.2197	100.00
4 4 0 .1	$\hat{\hat{Y}}_{SM}$	0.8778	100.76	5.1969	100.82
	$\bar{y}_{Rsym}$	1.6224	54.02	9.7475	54.01
	$\bar{y}_{Psym}$	1.8974	46.38	11.5884	45.48
	$\bar{y}_{sym}$	0.8849	100.00	5.2401	100.00
4 4 0.5	$\hat{\hat{Y}}_{SM}$	0.7059	123.73	4.1575	125.64
	$\bar{y}_{Rsym}$	1.0608	82.94	5.9926	87.23
	$\bar{y}_{Psym}$	2.5115	35.81	15.3753	34.23
	$\bar{y}_{sym}$	0.8857	100.00	5.2466	100.00
4 4 0.9	$\hat{\hat{Y}}_{SM}$	0.2746	297.65	1.7036	307.71
	$\bar{y}_{Rsym}$	0.3506	238.35	1.9912	264.64
	$\bar{y}_{Psym}$	3.1680	27.85	19.7950	27.24
	$\bar{y}_{sym}$	0.8618	100.00	5.3603	100.00

Table 3: MSE and PRE of estimator of $\bar{Y}$ for $\rho=(-0.9,\ -0.5,\ -0.1,\ 0.1,\ 0.5,\ 0.9)$ when $(\sigma^2_{Ysy},\ \sigma^2_{Xsy}=50,\ 40)$, $(\sigma^2_{Usy},\ \sigma^2_{Vsy}=4,\ 0)$ and $(k=4,\ 10)$

σ^2_{Usy} σ^2_{Vsy} ρ	Estimator	$k=4$		$k=10$	
4 0 -0.9		*MSE*	PRE	*MSE*	PRE

	$\hat{\bar{Y}}_{SM}$	0.2129	373.81	1.3611	392.37
	$\bar{y}_{Rsym}$	3.0382	29.63	19.5498	28.45
	$\bar{y}_{Psym}$	0.2712	302.95	1.5483	350.67
	$\bar{y}_{sym}$	0.8490	100.00	5.4510	100.00
4 0 - 0.5	$\hat{\bar{Y}}_{SM}$	0.6556	128.32	4.1526	129.66
	$\bar{y}_{Rsym}$	2.3718	37.21	15.1878	35.99
	$\bar{y}_{Psym}$	0.9555	90.49	5.6208	96.88
	$\bar{y}_{sym}$	0.8530	100.00	5.4068	100.00
4 0 -0.1	$\hat{\bar{Y}}_{SM}$	0.8389	100.44	5.2289	100.92
	$\bar{y}_{Rsym}$	1.8144	48.59	11.2343	47.34
	$\bar{y}_{Psym}$	1.5393	57.10	9.3650	56.75
	$\bar{y}_{sym}$	0.8269	100.00	5.2777	100.00
4 0 0 .1	$\hat{\bar{Y}}_{SM}$	0.8571	100.88	5.3197	100.92
	$\bar{y}_{Rsym}$	1.5313	57.34	9.3904	57.53
	$\bar{y}_{Psym}$	1.8033	48.85	11.2727	47.98
	$\bar{y}_{sym}$	0.8651	100.00	5.3695	100.00
4 0 0.5	$\hat{\bar{Y}}_{SM}$	0.6588	128.21	4.0573	129.53
	$\bar{y}_{Rsym}$	0.9474	91.13	5.5256	96.12
	$\bar{y}_{Psym}$	2.3569	37.45	14.9174	35.73
	$\bar{y}_{sym}$	0.8569	100.00	5.2801	100.00
4 0 0.9	$\hat{\bar{Y}}_{SM}$	0.2153	380.75	1.3312	391.23
	$\bar{y}_{Rsym}$	0.2678	312.84	1.4839	353.05
	$\bar{y}_{Psym}$	3.0759	29.66	19.2053	27.90
	$\bar{y}_{sym}$	0.8609	100.00	5.3232	100.00

Table 4: MSE and PRE of estimators of $\bar{Y}$ for $\rho = (-0.9,\ -0.5,\ -0.1,\ 0.1,\ 0.5,\ 0.9)$ when $(\sigma^2_{Ysy},\ \sigma^2_{Xsy} = 50,\ 40)$, $(\sigma^2_{Usy},\ \sigma^2_{Vsy} = 0,\ 4)$ and $(k = 4,\ 10)$

σ^2_U	σ^2_V	ρ	Estimator	$k=4$		$k=10$	
				MSE	PRE	*MSE*	PRE
0	4	-0.9	$\hat{\bar{Y}}_{SM}$	0.1526	526.32	1.2633	374.69
			$\bar{y}_{Rsym}$	3.0466	26.68	19.0479	25.40
			$\bar{y}_{Psym}$	0.2072	390.95	1.6083	300.74
			$\bar{y}_{sym}$	0.8031	100.00	4.8144	100.00
0	4	- 0.5	$\hat{\bar{Y}}_{SM}$	0.6019	133.33	3.7565	128.70
			$\bar{y}_{Rsym}$	2.2935	35.01	14.9476	32.47
			$\bar{y}_{Psym}$	0.8862	90.50	5.6185	86.19
			$\bar{y}_{sym}$	0.8025	100.00	4.8371	100.00
0	4	-0.1	$\hat{\bar{Y}}_{SM}$	0.8021	101.01	4.8819	100.90
			$\bar{y}_{Rsym}$	1.7638	45.67	11.4335	43.05
			$\bar{y}_{Psym}$	1.4864	54.20	9.5583	51.48
			$\bar{y}_{sym}$	0.8102	100.00	4.9258	100.00
0	4	0 .1	$\hat{\bar{Y}}_{SM}$	0.7904	101.01	4.8558	100.90
			$\bar{y}_{Rsym}$	1.4679	54.71	9.4814	51.54
			$\bar{y}_{Psym}$	1.7425	46.08	11.3524	43.07
			$\bar{y}_{sym}$	0.7984	100.00	4.8996	100.00
0	4	0.5	$\hat{\bar{Y}}_{SM}$	0.5920	133.33	3.8128	128.79
			$\bar{y}_{Rsym}$	0.8988	88.78	5.7078	85.83
			$\bar{y}_{Psym}$	2.3025	34.72	15.2434	32.24
			$\bar{y}_{sym}$	0.7894	100.00	4.9164	100.00
0	4	0.9	$\hat{\bar{Y}}_{SM}$	0.1472	526.32	1.3126	372.93
			$\bar{y}_{Rsym}$	0.2009	387.29	1.5973	305.94
			$\bar{y}_{Psym}$	2.9144	26.79	19.3430	25.58
			$\bar{y}_{sym}$	0.7745	100.00	4.9633	100.00

Table 5: MSE and PRE of estimators of $\bar{Y}$ for $\rho = (-0.9, -0.5, -0.1, 0.1, 0.5, 0.9)$ when $(\sigma^2_{Ysy}, \sigma^2_{Xsy} = 30, 20)$, $(\sigma^2_{Usy}, \sigma^2_{Vsy} = 0, 0)$ and $(k = 4, 10)$

σ^2_U σ^2_V ρ	Estimator	$k=4$		$k=10$	
		MSE	PRE	*MSE*	PRE
0 0 -0.9	$\hat{\hat{Y}}_{SM}$	0.0909	526.32	0.5607	526.32
	$\bar{y}_{Rsym}$	1.6506	29.33	10.4063	28.46
	$\bar{y}_{Psym}$	0.1152	411.94	0.6267	471.92
	$\bar{y}_{sym}$	0.4785	100.00	2.9508	100.00
0 0 - 0.5	$\hat{\hat{Y}}_{SM}$	0.3595	133.33	2.2281	133.33
	$\bar{y}_{Rsym}$	1.2658	37.60	8.1050	36.80
	$\bar{y}_{Psym}$	0.4914	95.59	2.8694	103.25
	$\bar{y}_{sym}$	0.4794	100.00	2.9707	100.00
0 0 -0.1	$\hat{\hat{Y}}_{SM}$	0.4734	101.01	2.9533	101.01
	$\bar{y}_{Rsym}$	0.9601	49.06	6.0552	49.05
	$\bar{y}_{Psym}$	0.8101	58.04	5.0133	59.17
	$\bar{y}_{sym}$	0.4782	100.00	2.9831	100.00
0 0 0 .1	$\hat{\hat{Y}}_{SM}$	0.4760	101.01	2.9300	101.01
	$\bar{y}_{Rsym}$	0.8088	58.08	4.9412	59.36
	$\bar{y}_{Psym}$	0.9580	49.10	5.9694	49.19
	$\bar{y}_{sym}$	0.4809	100.00	2.9596	100.00
0 0 0.5	$\hat{\hat{Y}}_{SM}$	0.3598	133.33	2.1916	133.33
	$\bar{y}_{Rsym}$	0.4924	95.52	2.8181	102.70
	$\bar{y}_{Psym}$	1.2717	37.55	7.9570	36.58
	$\bar{y}_{sym}$	0.4798	100.00	2.9221	100.00
0 0 0.9	$\hat{\hat{Y}}_{SM}$	0.0900	526.32	0.5454	526.32
	$\bar{y}_{Rsym}$	0.1139	412.48	0.6013	476.77
	$\bar{y}_{Psym}$	1.6369	29.29	10.0286	28.62
	$\bar{y}_{sym}$	0.4739	100.00	2.8703	100.00

Table 6: MSE and PRE of estimators of $\bar{Y}$ for $\rho = (-0.9, -0.5, -0.1, 0.1, 0.5, 0.9)$ when $(\sigma^2_{Ysy}, \sigma^2_{Xsy} = 30, 20)$, $(\sigma^2_{Usy}, \sigma^2_{Vsy} = 4, 4)$ and $(k = 4, 10)$

σ^2_U	σ^2_V	ρ	Estimator	$k=4$		$k=10$	
				MSE	PRE	*MSE*	PRE
4	4	-0.9	$\hat{\bar{Y}}_{SM}$	0.2141	245.25	1.3688	246.90
			$\bar{y}_{Rsym}$	1.8144	31.14	11.4683	30.21
			$\bar{y}_{Psym}$	0.2597	206.49	1.5499	220.54
			$\bar{y}_{sym}$	0.5469	100.00	3.4332	100.00
4	4	- 0.5	$\hat{\bar{Y}}_{SM}$	0.4411	119.61	2.7466	121.35
			$\bar{y}_{Rsym}$	1.4053	38.33	8.8619	37.87
			$\bar{y}_{Psym}$	0.6308	82.23	3.7145	89.05
			$\bar{y}_{sym}$	0.5356	100.00	3.3498	100.00
4	4	-0.1	$\hat{\bar{Y}}_{SM}$	0.5318	100.64	3.3815	100.71
			$\bar{y}_{Rsym}$	1.0957	48.21	6.8731	49.21
			$\bar{y}_{Psym}$	0.9466	55.50	5.8382	57.80
			$\bar{y}_{sym}$	0.5355	100.00	3.4061	100.00
4	4	0 .1	$\hat{\bar{Y}}_{SM}$	0.5264	100.65	3.2847	100.71
			$\bar{y}_{Rsym}$	0.9408	55.26	5.7992	56.61
			$\bar{y}_{Psym}$	1.0889	48.00	6.8231	48.25
			$\bar{y}_{sym}$	0.5300	100.00	3.3084	100.00
4	4	0.5	$\hat{\bar{Y}}_{SM}$	0.4512	119.66	2.7650	121.28
			$\bar{y}_{Rsym}$	0.6434	82.28	3.7461	88.76
			$\bar{y}_{Psym}$	1.4309	38.47	8.8873	37.94
			$\bar{y}_{sym}$	0.5478	100.00	3.3689	100.00
4	4	0.9	$\hat{\bar{Y}}_{SM}$	0.2135	245.10	1.3454	246.81
			$\bar{y}_{Rsym}$	0.2597	205.98	1.5098	221.54
			$\bar{y}_{Psym}$	1.8069	31.09	11.2391	30.20
			$\bar{y}_{sym}$	0.5439	100.00	3.3609	100.00

Table 7: MSE and PRE of various estimators of $\bar{Y}$ for $\rho = (-0.9,\ -0.5,\ -0.1,\ 0.1,\ 0.5,\ 0.9)$ when $(\sigma^2_{Ysy},\ \sigma^2_{Xsy} = 30,\ 20)$, $(\sigma^2_{Usy},\ \sigma^2_{Vsy} = 4,\ 0)$ and $(k = 4,\ 10)$

σ^2_U	σ^2_V	ρ	Estimator	$k=4$		$k=10$	
				MSE	PRE	*MSE*	PRE
4	0	-0.9	$\hat{\bar{Y}}_{SM}$	0.1549	339.80	0.9526	347.41
			$\bar{y}_{Rsym}$	1.7065	33.97	10.7264	31.86
			$\bar{y}_{Psym}$	0.1802	292.91	1.0181	325.85
			$\bar{y}_{sym}$	0.5424	100.00	3.3456	100.00
4	0	- 0.5	$\hat{\bar{Y}}_{SM}$	0.4170	126.20	2.6007	127.61
			$\bar{y}_{Rsym}$	1.3100	41.80	8.3943	40.10
			$\bar{y}_{Psym}$	0.5506	95.71	3.2318	102.76
			$\bar{y}_{sym}$	0.5346	100.00	3.3369	100.00
4	0	-0.1	$\hat{\bar{Y}}_{SM}$	0.5480	100.84	3.3158	100.87
			$\bar{y}_{Rsym}$	1.0240	53.96	6.2882	53.35
			$\bar{y}_{Psym}$	0.8729	62.93	5.2759	63.43
			$\bar{y}_{sym}$	0.5529	100.00	3.3453	100.00
4	0	0 .1	$\hat{\bar{Y}}_{SM}$	0.5392	100.83	3.2836	100.87
			$\bar{y}_{Rsym}$	0.8710	62.04	5.3157	62.30
			$\bar{y}_{Psym}$	1.0201	53.25	6.3381	52.36
			$\bar{y}_{sym}$	0.5440	100.00	3.3128	100.00
4	0	0.5	$\hat{\bar{Y}}_{SM}$	0.4231	126.32	2.6287	127.59
			$\bar{y}_{Rsym}$	0.5551	96.60	3.2768	102.24
			$\bar{y}_{Psym}$	1.3289	41.84	8.5503	39.82
			$\bar{y}_{sym}$	0.5429	100.00	3.3723	100.00
4	0	0.9	$\hat{\bar{Y}}_{SM}$	0.1554	338.22	0.9580	343.58
			$\bar{y}_{Rsym}$	0.1804	291.99	1.0122	325.38
			$\bar{y}_{Psym}$	1.6905	34.11	10.5726	32.11
			$\bar{y}_{sym}$	0.5386	100.00	3.3351	100.00

Table 8: MSE and PRE of estimators of $\bar{Y}$ for $\rho = (-0.9,\ -0.5,\ -0.1,\ 0.1,\ 0.5,\ 0.9)$ when $(\sigma^2_{Ysy},\ \sigma^2_{Xsy} = 30,\ 20)$, $(\sigma^2_{Usy},\ \sigma^2_{Vsy} = 0,\ 4)$ and $(k = 4,\ 10)$

σ^2_{Usy}	σ^2_{Vsy}	ρ	Estimator	$k=4$		$k=10$	
				MSE	PRE	*MSE*	PRE
0	4	-0.9	$\hat{\bar{Y}}_{SM}$	0.1506	311.51	0.9527	307.51
			$\bar{y}_{Rsym}$	1.7416	27.00	10.7147	27.35
			$\bar{y}_{Psym}$	0.1969	241.25	1.1215	263.37
			$\bar{y}_{sym}$	0.4825	100.00	2.9481	100.00
0	4	- 0.5	$\hat{\bar{Y}}_{SM}$	0.3821	124.39	2.3209	125.72
			$\bar{y}_{Rsym}$	1.3472	34.54	8.4743	34.29
			$\bar{y}_{Psym}$	0.5745	79.47	3.3109	86.94
			$\bar{y}_{sym}$	0.4768	100.00	2.9193	100.00
0	4	-0.1	$\hat{\bar{Y}}_{SM}$	0.4742	100.78	2.8995	100.82
			$\bar{y}_{Rsym}$	1.0438	44.29	6.4199	45.16
			$\bar{y}_{Psym}$	0.8936	51.55	5.4033	53.57
			$\bar{y}_{sym}$	0.4779	100.00	2.9231	100.00
0	4	0 .1	$\hat{\bar{Y}}_{SM}$	0.4750	100.78	2.8970	100.83
			$\bar{y}_{Rsym}$	0.8858	51.68	5.3981	53.62
			$\bar{y}_{Psym}$	1.0346	44.40	6.4255	45.12
			$\bar{y}_{sym}$	0.4787	100.00	2.9209	100.00
0	4	0.5	$\hat{\bar{Y}}_{SM}$	0.3749	124.30	2.3316	125.74
			$\bar{y}_{Rsym}$	0.5650	78.76	3.3139	86.82
			$\bar{y}_{Psym}$	1.3268	34.24	8.5098	34.11
			$\bar{y}_{sym}$	0.4676	100.00	2.9371	100.00
0	4	0.9	$\hat{\bar{Y}}_{SM}$	0.1487	309.32	0.9669	308.47
			$\bar{y}_{Rsym}$	0.1950	239.06	1.1360	264.43
			$\bar{y}_{Psym}$	1.7135	26.71	10.9902	27.13
			$\bar{y}_{sym}$	0.4728	100.00	3.0185	100.00

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